Outline

Supersolid Crystalline Helium 000 00 000 000000 0

Superfluidity and Supersolidity in ${}^{4}\mathrm{He}$

Author: Lars Bonnes Supervisor: Lode Pollet

Proseminar Theoretische Physik: Phase Transitions SS 07

18.06.2007

Superfluidity and Supersolidity in ${}^{4}\mathrm{He}$

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Outline

Superfluid Liquid Helium

Supersolid Crystalline Helium

Superfluid Liquid Helium

Motivation Two-fluid Model Condensation and Phase Transition Vortices

Summary

Supersolid Crystalline Helium

Kim-Chan Experiment Dependence on the Quality of the Crystal Communicating Vessels Theory Conclusion End

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Outline	Superfluid Liquid Helium ●0 ○○○○○○○○○○○○○○○○○○○○○○○○○○○○○○○○○○○	Supersolid Crystalline Helium 0000 00 000 000000 0 0
Motivation		

General Information on Helium:

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Motivation		

General Information on Helium:

• Stable isotopes: ${}^{3}\mathrm{He}$ and ${}^{4}\mathrm{He}$

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Motivation		

General Information on Helium:

- \blacktriangleright Stable isotopes: $^{3}\mathrm{He}$ and $^{4}\mathrm{He}$
- ▶ In nature: almost only ⁴He (99.999864%)

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Supersolid Crystalline Helium 000 00 000 000000 0

Motivation

General Information on Helium:

- \blacktriangleright Stable isotopes: ³He and ⁴He
- ▶ In nature: almost only ⁴He (99.999864%)
- Does not solidify at T = 0 for P < 26 bar</p>

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Outline

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Supersolid Crystalline Helium 0000 00 000 000000

Motivation



Superfluidity and Supersolidity in ⁴He

Outline

Superfluid Liquid Helium

Supersolid Crystalline Helium 000 00 000 000000 0

Two-fluid Model

Two-Fluid model by Tisza and Landau

Phenomenological model

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Superfluid Liquid Helium

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Two-fluid Model

Two-Fluid model by Tisza and Landau

- Phenomenological model
- Explains the frictionless flow of Helium

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Superfluid Liquid Helium

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Two-fluid Model

Two-Fluid model by Tisza and Landau

- Phenomenological model
- Explains the frictionless flow of Helium
- Gives quantitative predictions in the regime close to T = 0

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Two-fluid Model		
T=0		

- \blacktriangleright Liquid Helium with mass density ρ flowing through capillary with v
- ► T = 0

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Two-fluid Model		
T 0		

- \blacktriangleright Liquid Helium with mass density ρ flowing through capillary with v
- ► T = 0
- Energy density in the laboratory frame (LF): $E_{lf}^{He} = \frac{\rho}{2}v^2$

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Two fluid Model		
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T = 0

Friction will generate excitations (i.e. phonons).

Assume furtherly:

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Two-fluid Model		

Friction will generate excitations (i.e. phonons).

Assume furtherly:

T = 0

- One elementary excitation with momentum \vec{p} and energy density $\epsilon(\vec{p})$
- Energy in the rest frame (RF_{He}) of the Helium is $E_{rf}^{He} = \epsilon(\vec{p})$

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I wo-fluid Model		

T = 0

Friction will generate excitations (i.e. phonons).

Assume furtherly:

- ► One elementary excitation with momentum p
 and energy density

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)
- Energy in the rest frame (RF_{He}) of the Helium is $E_{rf}^{He} = \epsilon(\vec{p})$

► Apply Galilean transformation to
$$LF_{He}$$
:
 $E_{lf}^{He} = \epsilon(\vec{p}) + \vec{p}\vec{v} + \frac{\rho}{2}v^2$

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Limiting Velocity

$$E_{lf}^{He} = \underbrace{\epsilon(\vec{p}) + \vec{p}\vec{v}}_{\Delta E} + \frac{\rho v^2}{2}$$

Friction \longleftrightarrow Dissipation of energy from the Helium. This requires: $\Delta E < 0$

$$v > \frac{\epsilon(\vec{p})}{p}$$

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Two-fluid Model		

Limiting Velocity

$$v > \frac{\epsilon(\vec{p})}{p}$$

Necessary condition for the creation of an elementary excitation!

$$v < \frac{\epsilon(\vec{p})}{p} \iff$$
 no friction

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Two-fluid Model		

Drop assumption T = 0

What happens to the thermal excitations?

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Two-fluid Model		
$T \neq 0$		

> The criterion on creating new excitations remains valid.

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Superfluidity and Supersolidity in ${}^{4}\mathrm{He}$

Outline	Superfluid Liquid Helium	Supersolid Crystalline Helium 0000 00 000 000000 000000 0 0
Two-fluid Model		
$T \neq 0$		

- > The criterion on creating new excitations remains valid.
- Thermal excitations form a dilute gas of quasi particles (only valid close to T = 0)

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Two-fluid Model		
T eq 0		

- > The criterion on creating new excitations remains valid.
- ► Thermal excitations form a dilute gas of quasi particles (only valid close to T = 0)
- Gas ("COM") moves with \vec{v}' relative to the liquid

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Outline	Superfluid Liquid Helium 00 00000000000000 00000000 00000000	Supersolid Crystalline Helium 0000 000 000000 000000 0
Two-fluid Model		
T eq 0		

- The criterion on creating new excitations remains valid.
- ► Thermal excitations form a dilute gas of quasi particles (only valid close to T = 0)
- Gas ("COM") moves with \vec{v}' relative to the liquid
- ▶ In RF_{He} they are distributed via $n(\epsilon(\vec{p}) \vec{p}\vec{v}'; T)$



Momentum Density

Calculate **momentum density** \vec{P} :

$$\vec{P} = \int d au \vec{p} n(\epsilon - \vec{p} \vec{v})$$

Expand $n(\epsilon - \vec{p}\vec{v}') \approx n(\epsilon) - \vec{p}\vec{v}'\frac{dn}{d\epsilon} + \dots$

Average over spatial dimensions: $\vec{p}\vec{v'} \approx \frac{1}{3}p\hat{p}\vec{v'}$.

$$\vec{P} = \vec{v'} \int d\tau \frac{1}{3} p^2 \left(-\frac{dn(\epsilon)}{d\epsilon} \right) + \epsilon \vec{v} \cdot \vec{$$

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Outline

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Two-fluid Model

 $T \neq 0$

$$\vec{P} = \vec{v'} \int d\tau \frac{1}{3} p^2 \left(-\frac{dn(\epsilon)}{d\epsilon} \right)$$
$$\stackrel{\longleftrightarrow}{\overleftarrow{P} = \vec{v'} \rho_n}$$

Flow carries mass: $\rho_n = \int d\tau \frac{1}{3} p^2 \left(-\frac{dn(\epsilon)}{d\epsilon} \right)$

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Two-fluid Model		
$T \neq 0$		

The quasi particles may scatter with the capillary walls.

Energy can *dissipate* from the system.

The mass current will feel friction.

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Two-fluid Model		

The mass current experiencing friction has a mass density of ρ_n .

Two-fluid model: Write $\rho = \rho_s + \rho_n$.

• ρ_s : Density of superfluid Helium

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Two-fluid Model		

The mass current experiencing friction has a mass density of ρ_n .

Two-fluid model: Write $\rho = \rho_s + \rho_n$.

- ρ_s : Density of superfluid Helium
- ρ_n: Density of normal viscous Helium (gas of elementary excitations)

Outline	Superfluid Liquid Helium ○○ ○○○○○○○○○ ○○○○○○○○ ○	Supersolid Crystalline Helium 0000 00 000 000000 0 000000 0
Two-fluid Model		

The mass current experiencing friction has a mass density of ρ_n .

Two-fluid model: Write $\rho = \rho_s + \rho_n$.

- ρ_s : Density of superfluid Helium
- ρ_n: Density of normal viscous Helium (gas of elementary excitations)
- Both components interpenetrate eachother without interaction.

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Two-fluid Model		

Note:

The superfluid velocity is not an observable but the current

$$\vec{j} = \vec{v}\rho = \vec{v}_n\rho_n + \vec{v}_s\rho_s$$

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Two-fluid Model

Goal: Calculate ρ_n



Phonons: $\epsilon^{phonon}(p) = cp$ Rotons: $\epsilon^{roton}(p) = \Delta + \frac{(p-p_0)^2}{2m'}$ with $\Delta = 8.7K$ (B) (2) (C) (C)

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Two fluid Model		

Particle Distribution

Bose statistics: $n(\epsilon; T) = [e^{\epsilon/T} - 1]^{-1}$

Δ is large compared to T: approximate $n_{rot}(\epsilon; T)$ with **Boltzmann factor**:

 $n_{rot}(\epsilon; T) = e^{-\epsilon/T}$

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Two-fluid Model		

Results

Final results for
$$\rho_n = (\rho_n)_{phonon} + (\rho_n)_{roton}$$
:

$$(
ho_n)_{phonon} \propto T^4$$

 $(
ho_n)_{roton} \propto rac{1}{\sqrt{T}} e^{-rac{\Delta}{T}}$

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Two-fluid Model		

Behavior close to T_{λ} :

The Two-fluid model does not give any predictions.

Experimentally (e.g. Andronikahvili experiment):

$$\frac{\rho_n}{\rho} = \begin{cases} \left(\frac{T}{T_\lambda}\right)^{5.6}, & T < T_\lambda\\ 1, & T > T_\lambda \end{cases}$$

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Condensation and Phase Transition	Outline	Superfluid Liquid Helium ○○ ● ○○ ○○ ○○ ○○ ○○ ● ○○ ○○ ○○ ○○ ○	Supersolid Crystalline Helium 0000 00 000 000000 0 000000 0 0
Condensation and Phase Transition			
	Condensation and Phase Transition		

Condensation

Ideal Bose gas (IBG) of Helium atoms: $T_c = 3.3 K(\sim T_{\lambda})$.

Is there a connection between both phenomena?

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Outline

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Condensation and Phase Transition



Large quantitative differences!

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Outline

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Supersolid Crystalline Helium

Condensation and Phase Transition

Condensation in Superfluid Helium

Fundamental assumption: Superfluid Helium ($T < T_{\lambda}$) is characterized by gBEC.

gBEC in single-particle wave function: $\chi_0(\vec{r},t)$ with occupation number $N_0 \gg 1$

All other wave functions: Occupation number $\mathcal{O}(1)$

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Condensation and Phase Transition

Condensate Wave Function χ

Write
$$\chi_0(\vec{r},t) = \sqrt{n_0(\vec{r},t)}e^{i\phi(\vec{r},t)}$$

Particle density of condensate: $< n_0 >$

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Condensation and Rhose	Francision	

Ordering Parameter

Since we do not have a condensate above T_{λ} : $< n_0 >= 0$

Identify the particle density of the condensate as the **ordering parameter**!

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Condensation and Phase Transition

Ordering parameter

Ordering parameter: $< n_0 >$

Decays to 0 *smoothly* as $T \rightarrow T_{\lambda}$ thus we have an

second order phase transition!

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Condensation and Phase Transition

Condensate Density and Superfluid Density

It is very important that $n_0 \neq \rho_s!$

At T=0 we have $rac{
ho_s}{
ho}=1$ but $n_0pprox 10\%$

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Condensation and Phase Transition

Superfluid Velocity Potential Current: $\vec{j}_0 = \frac{\hbar}{2mi} (\chi_0^* \nabla \chi_0 - \chi_0 \nabla \chi_0^*)$ $\vec{j}_0 = \frac{\hbar}{2mi} (\chi_0^* \nabla \chi_0 - \chi_0 \nabla \chi_0^*)$

-

$$\vec{j}_0 = \frac{n}{m} n_0 \nabla \phi = n_0 \vec{v_s}$$

Define superfluid velocity:

$$v_s \equiv \frac{\hbar}{m} \nabla \phi$$

Velocity has a potential thus the flow is irrotational

$$abla imes v_s = 0$$

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Vortices

Onsager-Feynman quantization

By integration over closed loop:

$$\oint v_{s} dl = 2\pi n \frac{\hbar}{m}$$

- n = 0 in a simply connected region
- $n \neq 0$ contradicts $\nabla \times v_s = 0$
- Flow is quantized!

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Vortex

Macroscopilcal viewpoint: Vortex core is infinitesimally small and

- $\chi_0 = 0$ at the vortex line
- v_s diverges at the vortex line
- Define problem on a not simply connected region

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Vortices

Criterion for Appearance of Vortices

Consider

- \blacktriangleright Cylinder rotating with constant ω
- Energy in rotating frame: $E_{rot} = E M\omega$
- Calculate the energy cost for creating a vortex ΔE_V
- ▶ Calculate rotational energy of the superfluid component $M_s \omega$

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Vortices

Criterion for appearance of vortices

$$\Delta E_V = \int \frac{\rho_s v_s^2}{2} d^3 r = \mathbf{n}^2 L \rho_s \pi \left(\frac{h}{m}\right)^2 \ln \frac{R}{a}$$
$$\Delta E_{rot} = n M_s \omega = \mathbf{n} L \pi R^2 \frac{\hbar}{m} \rho_s \omega$$

 $\Delta E_{rot} \geq \Delta E_V$

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Vortices

Criterion for appearance of vortices

Critical angular velocity:

$$\omega_c = n^2 \frac{\hbar}{mR^2} ln \frac{R}{a}$$

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Vortices



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Vortices



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Vortices

Gedankenexperiment ("Hess-Fairbank")



- Helium in annular channel; rotating with $\omega < \omega_c$
- Not simply connected region. Superfluid flux is quantized!

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Vortices

Thought experiment ("Hess-Fairbank")

• $T > T_{\lambda}$: Normal velocity field $v_n = \omega r$

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Superfluid Liquid Helium

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Thought experiment ("Hess-Fairbank")

- $T > T_{\lambda}$: Normal velocity field $v_n = \omega r$
- ► Cool down below T_{λ} : Superfluid velocity can not be ωr

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Superfluid Liquid Helium

Supersolid Crystalline Helium 000 00 000 000000 0

Vortices

Thought experiment ("Hess-Fairbank")

- $T > T_{\lambda}$: Normal velocity field $v_n = \omega r$
- ► Cool down below T_{λ} : Superfluid velocity can not be ωr
- Define: $\omega' = \frac{\hbar}{mR^2}$ (n=1 in O.F.-quantization)

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Superfluid Liquid Helium

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Vortices

Thought experiment ("Hess-Fairbank")

- $T > T_{\lambda}$: Normal velocity field $v_n = \omega r$
- ► Cool down below T_{λ} : Superfluid velocity can not be ωr
- Define: $\omega' = \frac{\hbar}{mR^2}$ (n=1 in O.F.-quantization)
- Superfluid will rotate with $\omega_s = n\omega'$

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Superfluid Liquid Helium

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Vortices

Thought experiment ("Hess-Fairbank")

- $T > T_{\lambda}$: Normal velocity field $v_n = \omega r$
- ► Cool down below T_{λ} : Superfluid velocity can not be ωr
- Define: $\omega' = \frac{\hbar}{mR^2}$ (n=1 in O.F.-quantization)
- Superfluid will rotate with $\omega_s = n\omega'$
- *n* closest integer to $\frac{\omega}{\omega'}$

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Superfluid Liquid Helium

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Vortices

Thought experiment ("Hess-Fairbank")

- $T > T_{\lambda}$: Normal velocity field $v_n = \omega r$
- ► Cool down below T_{λ} : Superfluid velocity can not be ωr
- Define: $\omega' = \frac{\hbar}{mR^2}$ (n=1 in O.F.-quantization)
- Superfluid will rotate with $\omega_s = n\omega'$
- *n* closest integer to $\frac{\omega}{\omega'}$
- ω_s can be larger than ω (eventually observed 1967 by Hess and Fairbank)

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Superfluidity and Supersolidity in ${}^4\mathrm{He}$

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• At $T_{\lambda} = 2.17 \text{ K}$: Helium has a second order phase transition to a superfluid phase.

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- At $T_{\lambda} = 2.17 \, K$: Helium has a second order phase transition to a superfluid phase.
- ► Two-Fluids: Normal and superfluid component coexist; $\vec{j} = \vec{v}\rho = \vec{v}_n\rho_n + \vec{v}_s\rho_s$
- T = 0: Only $\approx 10\%$ of Helium in condensate.



- At $T_{\lambda} = 2.17 \, K$: Helium has a second order phase transition to a superfluid phase.
- ► Two-Fluids: Normal and superfluid component coexist; $\vec{j} = \vec{v}\rho = \vec{v}_n\rho_n + \vec{v}_s\rho_s$
- T = 0: Only $\approx 10\%$ of Helium in condensate.
- Circulation of superfluid is quantized (Onsager-Feynman)

Superfluid Liquid Helium

Supersolid Crystalline Helium

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Superfluid Liquid Helium

Supersolid Crystalline Helium

Hexagonal Close Packing



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Superfluid Liquid Helium

Supersolid Crystalline Helium

Kim-Chan Experiment

Kim-Chan Experiment



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Superfluid Liquid Helium

Supersolid Crystalline Helium

Kim-Chan Experiment

Kim-Chan Experiment

Nonclassical Rotational Inertia Fraction:

$$NCRIF := \frac{\Delta I}{I_c}$$

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Superfluid Liquid Helium

Supersolid Crystalline Helium

Kim-Chan Experiment

Kim-Chan Experiment

Nonclassical Rotational Inertia Fraction



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Outline	Superfluid Liquid Helium 00 00000000000000 00000000 00000000 0	Supersolid Crystalline Helium 000 00 000 000 000000 000000

Kim-Chan Experiment

New Phase Diagram



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Superfluid Liquid Helium

Supersolid Crystalline Helium

Dependence on the Quality of the Crystal

Quality of the Crystal

Experiments by Rittner and Reppy in 2006:

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Similar experiment as Kim-Chan.

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- Start with one solid sample and look for NCRIF.

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- Similar experiment as Kim-Chan.
- Start with one solid sample and look for NCRIF.
- Anneal the crystal.

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Superfluid Liquid Helium

Supersolid Crystalline Helium

Dependence on the Quality of the Crystal

Quality of the Crystal

Experiments by Rittner and Reppy in 2006:

- Similar experiment as Kim-Chan.
- Start with one solid sample and look for NCRIF.
- Anneal the crystal.
- Measure NCRIF.
- etc.

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Superfluid Liquid Helium

Dependence on the Quality of the Crystal

Quality of the Crystal

Results:

- ▶ Reproduce NCRIF as in Kim-Chan experiment.
- By annealing NCRIF could be suppressed.
- "[...]the superfluid signal is not an universal property of solid ⁴He"

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Superfluid Liquid Helium

Supersolid Crystalline Helium

Communicating Vessels

Communicating Vessels $T \approx 50 \, mK$



Superfluidity and Supersolidity in ${}^{4}\mathrm{He}$

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Outline

Superfluid Liquid Helium

Supersolid Crystalline Helium

Communicating Vessels

Communicating Vessels $T \approx 50 \, mK$



Superfluidity and Supersolidity in

Outline	Superfluid Liquid Helium 00 00000000000000 00000000 000000000 0	Supersolid Crystalline Helium ○○○○ ○○● ○○○○○○ ○○○○○○○○○○○○○○○○○○○○○
Communicating Vessels		

Problem

- No quantitative description of crystal quality.
- Behaviour of "history" of the crystal.
- One found: Annealing can also increase NCRIF.

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Outline	Superfluid Liquid Helium 00 00000000000000 00000000 00000000 0	Supersolid Crystalline Helium
Theory		

Explanation

- Homogeneous scenario: Can a crystal on a perfect lattice be not insulating?
- Inhomogeneous scenario: Is this effect resulting from defects of the crystal?

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Outline	Superfluid Liquid Helium 00 000000000000000000000000 000000000	Supersolid Crystalline Helium ○○○○ ○○○ ○●○○○○ ○
Theory		

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Superfluidity and Supersolidity in ${}^4\mathrm{He}$

Outline	Superfluid Liquid Helium 00 000000000000000000000000 000000000	Supersolid Crystalline Helium ○○○ ○○ ○●○ ○●○○○○ ○
Theory		

 Ground state: Classical crystal features integer number of atoms per unit cell

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Outline	Superfluid Liquid Helium 00 000000000000000 00000000 00000000 0	Supersolid Crystalline Helium ○○○○ ○○○ ○●○○○○ ○
Theory		

- Ground state: Classical crystal features integer number of atoms per unit cell
- Vacancies or interstitials in the crystal may be present in the true ground state of Helium

Outline	Superfluid Liquid Helium 00 000000000000000000000000 000000000	Supersolid Crystalline Helium ○○○○ ○○○ ○●○○○○ ○●○○○○ ○
Theorem		
Theory		

- Ground state: Classical crystal features integer number of atoms per unit cell
- Vacancies or interstitials in the crystal may be present in the true ground state of Helium
- Highly mobile vacancies may give rise to supersolid effects

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Superfluid Liquid Helium

Supersolid Crystalline Helium

Theory

True Ground State of Helium

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Superfluidity and Supersolidity in ${}^{4}\mathrm{He}$

Outline	Superfluid Liquid Helium 00 000000000000000 00000000 00000000 0	Supersolid Crystalline Helium
Theory		

True Ground State of Helium

 Ground state will have vacancies / interstitials if their creation does not cost energy E_{gap} (gapless vacancies)

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Outline	Superfluid Liquid Helium 00 000000000000000000 00000000 0000000	Supersolid Crystalline Helium
Theory		

True Ground State of Helium

- Ground state will have vacancies / interstitials if their creation does not cost energy E_{gap} (gapless vacancies)
- Use quantum Monte Carlo to measure $n(\vec{r}, \vec{r'})$ and E_{gap}

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Outline

Superfluid Liquid Helium

Supersolid Crystalline Helium

Theory

Monte Carlo Results



No ODLRO.

Superfluidity and Supersolidity in ${}^{4}\mathrm{He}$

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Outline	Superfluid Liquid Helium 00 000000000000000 0000000 00000000 0	Supersolid Crystalline Helium
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Theory

True Ground State of Helium

Since large energy gap and no ODLRO:

The true ground state of solid Helium is a commensurate hcp crystal.

Superfluidity and Supersolidity in ${}^{4}\mathrm{He}$

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Outline	Superfluid Liquid Helium 00 00000000000000 0000000 00000000 0	Supersolid Crystalline Helium ○○○ ○○○ ○○○○ ○○○○ ○○○○ ○○○○ ○
Theory		

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Superfluidity and Supersolidity in ${}^4\mathrm{He}$

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Outline	Superfluid Liquid Helium 00 000000000000000 0000000 00000000 0	Supersolid Crystalline Helium ○○○○ ○○○ ○○○○ ○○○○● ○
Theory		

• Dislocations, grain boundaries etc.

Outline	Superfluid Liquid Helium 00 000000000000000 00000000 00000000 0000	Supersolid Crystalline Helium ○○○ ○○ ○○○ ○○○ ○○○ ○○○ ○○○ ○○
Theory		

- Dislocations, grain boundaries etc.
- Specific type of defect or influence on NCRIF are known yet.

Outline	Superfluid Liquid Helium 00 000000000000000000000000 00000000 0000	Supersolid Crystalline Helium
Theony		

- Dislocations, grain boundaries etc.
- Specific type of defect or influence on NCRIF are known yet.
- Superfluid Helium flowing through channels. (grain size $\sim 10 \text{ nm}$)

Outline	Superfluid Liquid Helium 00 000000000000000 00000000 00000000 0	Supersolid Crystalline Helium
Conclusion		

Conclusion

Superfluidity and Supersolidity in ${}^4\mathrm{He}$

Outline	Superfluid Liquid Helium 00 000000000000000 00000000 00000000 0000	Supersolid Crystalline Helium
Conclusion		

Conclusion

Shift in resonance period of rotating solid Helium gives rise to supersolid behavior.

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Outline	Superfluid Liquid Helium 00 000000000000000000000000000000000	Supersolid Crystalline Helium
Conclusion		
Conclusion		

Conclusion

- Shift in resonance period of rotating solid Helium gives rise to supersolid behavior.
- Strong dependence on crystal quality.
- Not a universal property of Helium perfect helium crystal is insulating.

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Outline	Superfluid Liquid Helium	Supersolid Crystalline Helium
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End

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Superfluidity and Supersolidity in ${}^4\mathrm{He}$

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