Superfluidity and Supersolidity in $^4\mathrm{He}$

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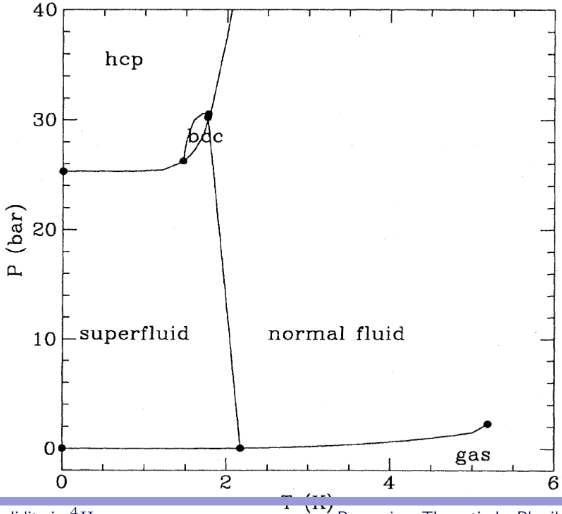
- Stable isotopes: $^3\mathrm{He}$ and $^4\mathrm{He}$
- In nature: almost only $^4\mathrm{He}$ (99.999864%)
- Does not solidify at $T = 0$ for $P < 26\ bar$

Motivation



Two-Fluid model by Tisza and Landau

- Phenomenological model

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- Explains the frictionless flow of Helium

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- Phenomenological model
- Explains the frictionless flow of Helium
- Gives quantitative predictions in the regime close to $T = 0$

T = 0

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- Liquid Helium with mass density ρ flowing through capillary with v
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- Energy density in the laboratory frame (LF): $E_{lf}^{He} = \frac{\rho}{2} v^2$

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- One elementary excitation with momentum $\vec{p}$ and energy density $\epsilon(\vec{p})$
- Energy in the rest frame (RF_{He}) of the Helium is $E_{rf}^{He} = \epsilon(\vec{p})$

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Assume furtherly:

- One elementary excitation with momentum $\vec{p}$ and energy density $\epsilon(\vec{p})$
- Energy in the rest frame (RF_{He}) of the Helium is $E_{rf}^{He} = \epsilon(\vec{p})$
- Apply Galilean transformation to LF_{He}:
  $E_{lf}^{He} = \epsilon(\vec{p}) + \vec{p}\vec{v} + \frac{\rho}{2}v^2$

Limiting Velocity

$$E_{lf}^{He} = \underbrace{\epsilon(\vec{p}) + \vec{p}\vec{v}}_{\Delta E} + \frac{\rho v^2}{2}$$

Friction $\longleftrightarrow$ Dissipation of energy from the Helium.
This requires: $\Delta E < 0$

$$v > \frac{\epsilon(\vec{p})}{p}$$

Limiting Velocity

$$v > \frac{\epsilon(\vec{p})}{p}$$

Necessary condition for the creation of an elementary excitation!

$$v < \frac{\epsilon(\vec{p})}{p} \Longleftrightarrow \textbf{no friction}$$

Drop assumption $T = 0$

What happens to the thermal excitations?

$T \neq 0$

Assume:

- The criterion on creating new excitations remains valid.

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- Thermal excitations form a dilute gas of quasi particles (only valid close to $T = 0$)
- Gas ("COM") moves with $\vec{v}'$ relative to the liquid
- In RF_{He} they are distributed via $n(\epsilon(\vec{p}) - \vec{p}\vec{v}'; T)$

Momentum Density

Calculate **momentum density** $\vec{P}$:

$$\vec{P} = \int d\tau \vec{p} n(\epsilon - \vec{p}\vec{v})$$

Expand $n(\epsilon - \vec{p}\vec{v}') \approx n(\epsilon) - \vec{p}\vec{v'} \frac{dn}{d\epsilon} + \ldots$

Average over spatial dimensions: $\vec{p}\vec{v'} \approx \frac{1}{3} p \hat{p} \vec{v'}$.

$$\vec{P} = \vec{v'} \int d\tau \frac{1}{3} p^2 \left(-\frac{dn(\epsilon)}{d\epsilon} \right)$$

$T \neq 0$

$$\vec{P} = \vec{v'} \int d\tau \frac{1}{3} p^2 \left(-\frac{dn(\epsilon)}{d\epsilon} \right)$$

$$\longleftrightarrow$$

$$\vec{P} = \vec{v'} \rho_n$$

Flow carries mass: $\rho_n = \int d\tau \frac{1}{3} p^2 \left(-\frac{dn(\epsilon)}{d\epsilon} \right)$

$T \neq 0$

The quasi particles may scatter with the capillary walls.

$$\longleftrightarrow$$

Energy can *dissipate* from the system.

$$\longleftrightarrow$$

The mass current will feel *friction*.

Two-fluid model

The mass current experiencing friction has a mass density of ρ_n.

Two-fluid model: Write $\rho = \rho_s + \rho_n$.

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- ρ_s: Density of superfluid Helium
- ρ_n: Density of normal viscous Helium (gas of elementary excitations)
- Both components interpenetrate eachother *without interaction*.

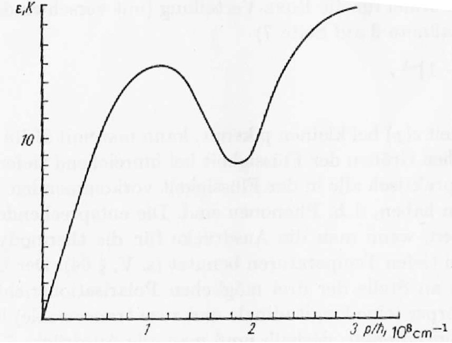
Two-fluid model

Note:

The superfluid velocity is not an observable but the current

$$\vec{j} = \vec{v}\rho = \vec{v}_n\rho_n + \vec{v}_s\rho_s$$

Goal: Calculate ρ_n



Phonons: $\epsilon^{phonon}(p) = cp$

Rotons: $\epsilon^{roton}(p) = \Delta + \frac{(p-p_0)^2}{2m'}$ with $\Delta = 8.7\,K$

Particle Distribution

Bose statistics: $n(\epsilon; T) = [e^{\epsilon/T} - 1]^{-1}$

Δ is large compared to T: approximate $n_{rot}(\epsilon; T)$ with **Boltzmann factor**:
$n_{rot}(\epsilon; T) = e^{-\epsilon/T}$

Results

Final results for $\rho_n = (\rho_n)_{phonon} + (\rho_n)_{roton}$:

$$(\rho_n)_{phonon} \propto T^4$$

$$(\rho_n)_{roton} \propto \frac{1}{\sqrt{T}} e^{-\frac{\Delta}{T}}$$

Behavior close to T_λ:
The Two-fluid model does not give any predictions.

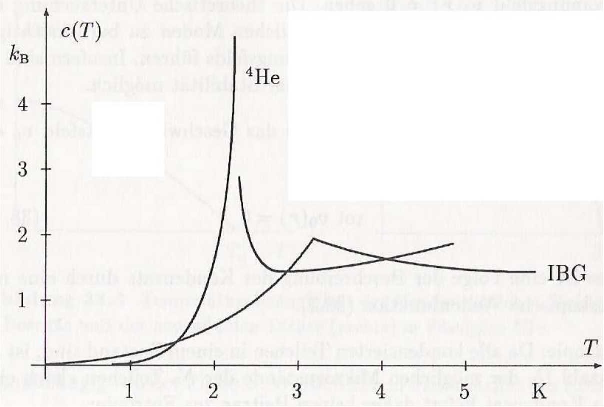
Experimentally (e.g. Andronikahvili experiment):

$$\frac{\rho_n}{\rho} = \begin{cases} \left(\frac{T}{T_\lambda}\right)^{5.6}, & T < T_\lambda \\ 1, & T > T_\lambda \end{cases}$$

Condensation

Ideal Bose gas (IBG) of Helium atoms: $T_c = 3.3\,K (\sim T_\lambda)$.

Is there a connection between both phenomena?



Large quantitative differences!

Condensation in Superfluid Helium

Fundamental assumption: Superfluid Helium ($T < T_\lambda$) is characterized by gBEC.

gBEC in single-particle wave function: $\chi_0(\vec{r}, t)$ with occupation number $N_0 \gg 1$

All other wave functions: Occupation number $\mathcal{O}(1)$

Condensate Wave Function χ

Write $\chi_0(\vec{r}, t) = \sqrt{n_0(\vec{r}, t)}e^{i\phi(\vec{r},t)}$

Particle density of condensate: $< n_0 >$

Ordering Parameter

Since we do not have a condensate above T_λ: $< n_0 >= 0$

Identify the particle density of the condensate as the **ordering parameter**!

Ordering parameter

Ordering parameter: $< n_0 >$

Decays to 0 *smoothly* as $T \to T_\lambda$ thus we have an

second order phase transition!

Condensate Density and Superfluid Density

It is very important that $n_0 \neq \rho_s$!

At $T = 0$ **we have**
$\frac{\rho_s}{\rho} = 1$ but
$n_0 \approx 10\%$

Superfluid Velocity Potential

Current: $\vec{j}_0 = \frac{\hbar}{2mi}(\chi_0^* \nabla \chi_0 - \chi_0 \nabla \chi_0^*)$

$$\vec{j}_0 = \frac{\hbar}{m} n_0 \nabla \phi = n_0 \vec{v_s}$$

Define **superfluid velocity**:

$$v_s \equiv \frac{\hbar}{m} \nabla \phi$$

Velocity has a potential thus the flow is irrotational

$$\nabla \times v_s = 0$$

Onsager-Feynman quantization

By integration over closed loop:

$$\oint v_s dl = 2\pi n \frac{\hbar}{m}$$

- $n = 0$ in a simply connected region
- $n \neq 0$ contradicts $\nabla \times v_s = 0$
- Flow is quantized!

Vortex

Macroscopilcal viewpoint: Vortex core is infinitesimally small and

- $\chi_0 = 0$ at the vortex line
- v_s diverges at the vortex line
- Define problem on a not simply connected region

Criterion for Appearance of Vortices

Consider

- Cylinder rotating with constant ω
- Energy in rotating frame: $E_{rot} = E - M\omega$
- Calculate the energy cost for creating a vortex ΔE_V
- Calculate rotational energy of the superfluid component $M_s\omega$

Criterion for appearance of vortices

$$\Delta E_V = \int \frac{\rho_s v_s^2}{2} d^3 r = \mathbf{n^2} L \rho_s \pi \left(\frac{h}{m}\right)^2 \ln \frac{R}{a}$$

$$\Delta E_{rot} = n M_s \omega = \mathbf{n} L \pi R^2 \frac{\hbar}{m} \rho_s \omega$$

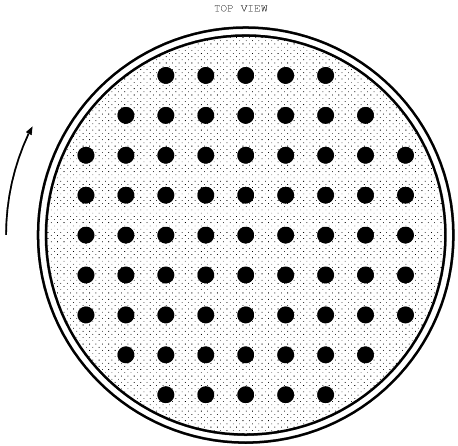
$$\Delta E_{rot} \geq \Delta E_V$$

Criterion for appearance of vortices

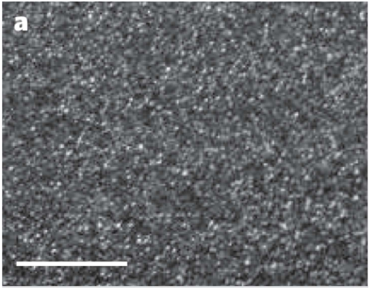
Critical angular velocity:

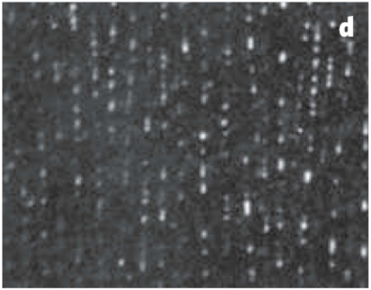
$$\omega_c = n^2 \frac{\hbar}{mR^2} ln \frac{R}{a}$$

Vortices

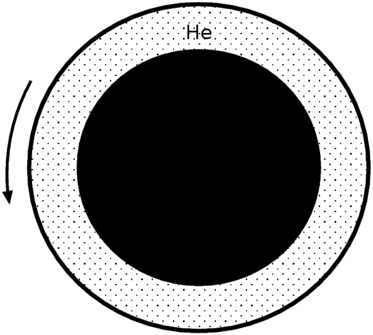


Vortices





Gedankenexperiment ("Hess-Fairbank")



- Helium in annular channel; rotating with $\omega < \omega_c$
- Not simply connected region. Superfluid flux is quantized!

Thought experiment ("Hess-Fairbank")

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- n closest integer to $\frac{\omega}{\omega'}$
- ω_s can be larger than ω (eventually observed 1967 by Hess and Fairbank)

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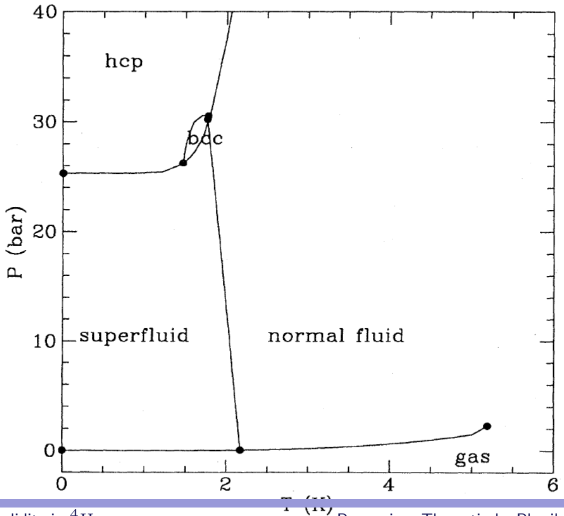
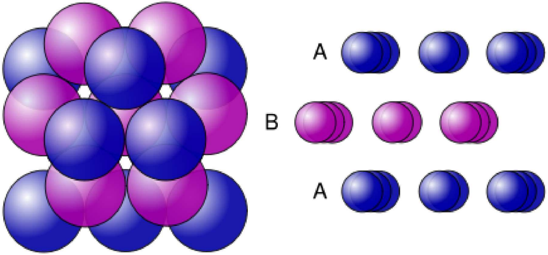
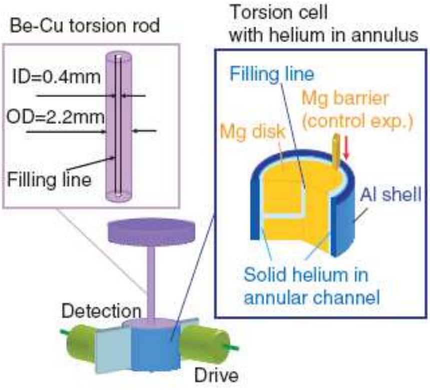
- Two-Fluids: Normal and superfluid component coexist; $\vec{j} = \vec{v}\rho = \vec{v}_n\rho_n + \vec{v}_s\rho_s$
- $T = 0$: Only $\approx 10\%$ of Helium in condensate.
- Circulation of superfluid is quantized (Onsager-Feynman)


hcp
bcc
superfluid
normal fluid
gas
P (bar)
T (K)
0
10
20
30
40
0
2
4
6

Hexagonal Close Packing



Kim-Chan Experiment



Kim-Chan Experiment

Nonclassical Rotational Inertia Fraction:

$$NCRIF := \frac{\Delta I}{I_c}$$

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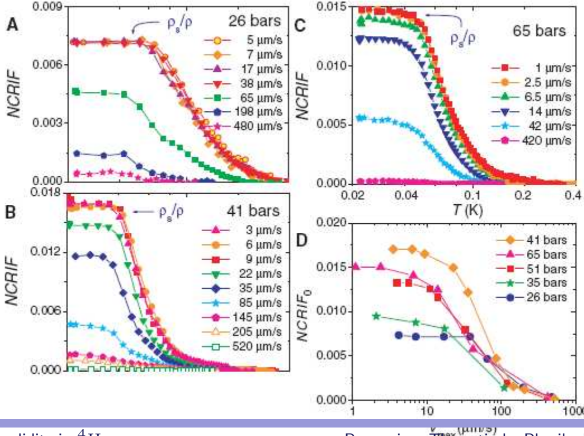
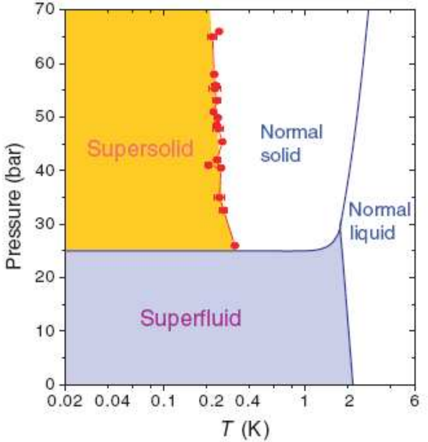


New Phase Diagram



Quality of the Crystal

Experiments by Rittner and Reppy in 2006:

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Experiments by Rittner and Reppy in 2006:

- Similar experiment as Kim-Chan.
- Start with one solid sample and look for NCRIF.
- Anneal the crystal.
- Measure NCRIF.
- etc.

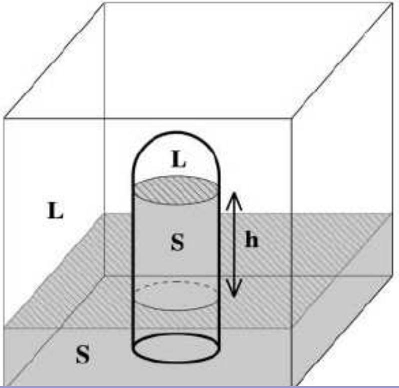
Quality of the Crystal

Results:

- Reproduce NCRIF as in Kim-Chan experiment.
- By annealing NCRIF could be suppressed.
- **“[...]the superfluid signal is not an universal property of solid ^{4}He”**

Communicating Vessels

$T \approx 50\,mK$



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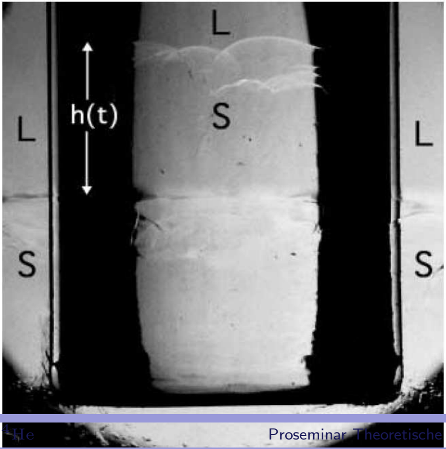

Problem

- No quantitative description of crystal quality.
- Behaviour of "history" of the crystal.
- One found: Annealing can also increase NCRIF.

Explanation

- Homogeneous scenario: Can a crystal on a perfect lattice be not insulating?
- Inhomogeneous scenario: Is this effect resulting from defects of the crystal?

Vacancies

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- Ground state: Classical crystal features integer number of atoms per unit cell
- Vacancies or interstitials in the crystal may be present in the true ground state of Helium
- Highly mobile vacancies may give rise to supersolid effects

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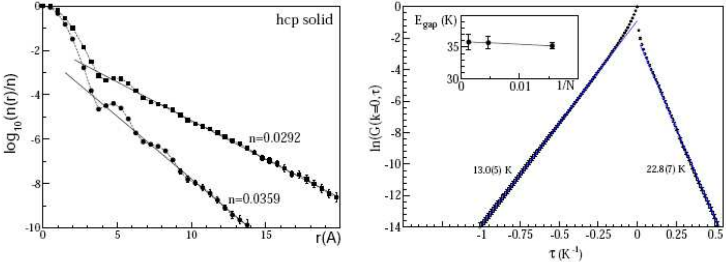
- Use quantum Monte Carlo to measure $n(\vec{r}, \vec{r'})$ and E_{gap}

Monte Carlo Results



No ODLRO.

True Ground State of Helium

Since large energy gap and no ODLRO:

The true ground state of solid Helium is a commensurate hcp crystal.

Macroscopical Models

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- Superfluid Helium flowing through channels. (grain size $\sim 10\,nm$)

Conclusion

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- Shift in resonance period of rotating solid Helium gives rise to supersolid behavior.

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- Shift in resonance period of rotating solid Helium gives rise to supersolid behavior.
- Strong dependence on crystal quality.
- Not a universal property of Helium - perfect helium crystal is insulating.

End...