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# *The Phases of Quantum Chromodynamics*

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# Overview


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QFT and thermodynamics

QCD and symmetries

Simplifications of QCD

## The QCD Phase Diagram

Low temperature and finite density — The ground state $T = 0$

Low temperature and finite density — The situation for low temperatures

Quark-gluon-plasma at high temperatures

High temperature and large chemical potential

## Colour superconductivity and colour flavour locking

Symmetry breaking due to colour superconductivity

Physical consequences of CFL

## Relativistic Heavy Ion Collisions

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# Motivation and methods

- Motivation:
  - High-temperature universe, i.e. fractions of a second after the Big Bang
  - High-density matter, i.e. in a neutron star
  - Understanding of QCD in extreme environments $\rightarrow$ deeper understanding of theory

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  - High-temperature universe, i.e. fractions of a second after the Big Bang
  - High-density matter, i.e. in a neutron star
  - Understanding of QCD in extreme environments $\longrightarrow$ deeper understanding of theory
- Methods:
  - Almost no rigorous results
  - High temperatures, low densities: lattice calculations
  - High densities: analytic calculations

# Finite temperature QFT

Short repetition of what has already been said for confinement/deconfinement:

- Partition function of a statistical system ($\beta = 1/T$):

$$Z = \sum_{\text{all states}} e^{-\beta E}$$

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  and Euclidian time $\tau = it$



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- Rewrite using

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  and Euclidian time $\tau = it$

- Final result:

$$Z = \int_{\phi(0)=\phi(\beta)} \mathcal{D}\phi \ \exp\left[ - \int_0^\beta d\tau \int d^3x \mathcal{L}_E \right]$$

# Chemical potential

- In canonical ensemble: particle number $N$ kept constant

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- In canonical ensemble: particle number $N$ kept constant
- In a field theory:
  - Creation of particle-antiparticle pairs; conserved quantity defined through number of particles minus number of antiparticles
- More convenient: allow particle number to fluctuate, but introduce weight factor similar to Gibbs factor for the energy

# Grand canonical ensemble

- Grand canonical ensemble: partition function defined by

$$Z = \sum_{\text{all states } \alpha} e^{-\beta E_\alpha} e^{\beta \mu N_\alpha}$$


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- $\mu$ is called the chemical potential associated with the charge $N$: energy change due to the introduction of particles to the system
- Potentially many chemical potentials in a system; here, baryon chemical potential $\mu_B$ used, associated with baryon number $N_B$
- Grand (canonical) potential = Helmholtz free energy:

$$\Omega(T, \mu) = -T \ln Z = -pV$$

- Minimzed in equilibirum $\rightarrow$ $p$ maximized

# Variables in the phase diagram

- Need to define driving parameters of the phase diagram
- Choose such that they are constant throughout the system even at phase coexistence and intensive
- One obvious variable: temperature $T$
- Density? No, because at phase coexistence, different density in different phases
- Chemical potential: connected to density, but constant across phase boundaries

# Chiral symmetry: conventions

- Fermionic field described by Dirac equation
- Weyl representation used:

$$\gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \psi(x) = \begin{pmatrix} \psi_L(x)' \\ \psi_R(x)' \end{pmatrix}$$

$$\begin{aligned} \psi_L(x) &= \frac{1-\gamma^5}{2}\psi(x) \\ \psi_R(x) &= \frac{1+\gamma^5}{2}\psi(x). \end{aligned}$$

- $\psi'_L$ and $\psi'_R$: left-handed and right-handed components

# Chiral symmetry: conserved currents

- Consider symmetry transformations:

$$U(1)_B : \psi(x) \rightarrow e^{i\alpha}\psi(x)$$
$$\psi(x) \rightarrow e^{i\alpha\gamma^5}\psi(x)$$



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- Associated currents:

$$\begin{aligned} j^\mu(x) &= \overline{\psi}(x)\gamma^\mu\psi(x) \\ j^{\mu 5}(x) &= \overline{\psi}(x)\gamma^\mu\gamma^5\psi(x) \end{aligned}$$



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- Divergences:

$$\begin{aligned} \partial_\mu j^\mu &= 0 \\ \partial_\mu j^{\mu 5} &= 2im\overline{\psi}\gamma^5\psi \end{aligned}$$

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- Divergences:

$$\partial_\mu j^\mu = 0$$
$$\partial_\mu j^{\mu 5} = 2im\overline{\psi}\gamma^5\psi$$

- The associated currents are conserved!

# Chiral symmetry: Left- and right-handed currents



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- Define left- and right-handed currents

$$\begin{aligned} j_L^\mu &= \overline{\psi_L}\gamma^\mu\psi_L \\ j_R^\mu &= \overline{\psi_R}\gamma^\mu\psi_R \end{aligned}$$

- We find:

$$\begin{aligned} j_L^\mu + j_R^\mu &= j^\mu \\ \partial_\mu j_L^\mu = \partial_\mu j_R^\mu &= 0. \end{aligned}$$

- Currents for left- and right-handed quarks are conserved separately!

# Chiral symmetry in QCD

- Consider QCD with a doublet of massless quark flavours:

$$Q = \begin{pmatrix} u \\ d \end{pmatrix} = Q_L + Q_R, \quad Q_{L/R} = \frac{1 \mp \gamma^5}{2} \begin{pmatrix} u \\ d \end{pmatrix}$$



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- Transform separately under isospin transformations

$$U_L, U_R \in SU(2): \; Q_L \to U_L Q_L, \; Q_R \to U_R Q_R$$

- Chiral flavour symmetry of QCD: $SU(N_f)_L \times SU(N_f)_R$, $N_f$ number of (massless) quark flavours



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- Chiral flavour symmetry of QCD: $SU(N_f)_L \times SU(N_f)_R$, $N_f$ number of (massless) quark flavours
- Full symmetry group of QCD:

$$SU(3)_C \times SU(N_f)_L \times SU(N_f)_R \times U(1)_B$$

# Chiral symmetry breaking in QCD

- Strong attractive interactions between quarks and antiquarks, i.e. negative contribution to total energy
- Energy cost to create a pair of massless particles is very small



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# Chiral symmetry breaking in QCD

- Strong attractive interactions between quarks and antiquarks, i.e. negative contribution to total energy
- Energy cost to create a pair of massless particles is very small
- Therefore, vacuum is populated by quark-antiquark pairs
- These have overall momentum and angular momentum of 0 $\longrightarrow$ they carry net helicity charge, pairs of left-handed quarks and left-handed antiquark, which is antiparticle of right-handed quark

# Chiral symmetry breaking in QCD 2



- Non-zero expectation value for quark-antiquark pairs:

$$\langle 0|\overline{Q}Q|0\rangle = \langle 0|\overline{Q_L}Q_R + \overline{Q_R}Q_L|0\rangle \neq 0$$

- Apply chiral flavour symmetries $U_L, U_R \in SU(2)$:

$$\langle 0|\overline{Q_L}Q_R + \overline{Q_R}Q_L|0\rangle = \langle 0|\overline{Q_L}U_L^\dagger U_R Q_R + \overline{Q_R}U_R^\dagger U_L Q_L|0\rangle$$

# Chiral symmetry breaking in QCD 2



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$$\langle 0|\overline{Q_L}Q_R + \overline{Q_R}Q_L|0\rangle = \langle 0|\overline{Q_L}U_L^\dagger U_R Q_R + \overline{Q_R}U_R^\dagger U_L Q_L|0\rangle$$

- Only fulfilled for $U_L = U_R$!
- Appearance of condensate breaks $SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$
- Spontaneous symmetry breaking: symmetry of Lagrangian not realized in ground state

# Goldstone theorem

- Assumptions:
  - No long-range interactions, e.g. Coulomb forces $\longrightarrow$ true for QCD w/o Coulomb
  - Lagrangian has a continuous, global symmetry
  - Potential term selects a ground state (minimum potential)
  - Ground state does not respect symmetry



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  - Lagrangian has a continuous, global symmetry
  - Potential term selects a ground state (minimum potential)
  - Ground state does not respect symmetry
- Consequences (proof of classical, scalar case can be found in the report):
  - Occurence of a massless bosonic particle called (Nambu-)Goldstone boson
  - True also for non-classical theories

# Higgs mechanism

- Goldstone theorem: spontaneous breaking of a global continuous symmetry
- Higgs mechanism: spontaneous breaking of a local gauge symmetry!

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- Assumptions:
  - Lagrange function with local gauge symmetry
  - Ground state which is not invariant under gauge transformations



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- Goldstone theorem: spontaneous breaking of a global continuous symmetry
- Higgs mechanism: spontaneous breaking of a local gauge symmetry!
- Assumptions:
  - Lagrange function with local gauge symmetry
  - Ground state which is not invariant under gauge transformations
- Consequences:
  - New term in the Lagrangian: $\Delta\mathcal{L} = \frac{1}{2} m_A^2 A_\mu A^\mu$
  - Gauge bosons, described by $A_\mu$, acquire mass!

# Simplified QCD

- Electroweak interactions are ignored
- Two massless quarks $u$ and $d$, no other quarks
- Leads to global $SU(2)_L \times SU(2)_R \times U(1)_B$ symmetry, broken down to $SU(2)_V \times U(1)_B$



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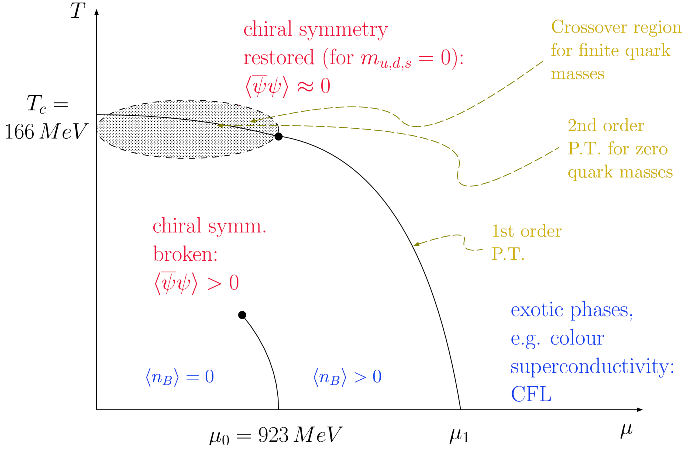
# The QCD Phase Diagram — schematically



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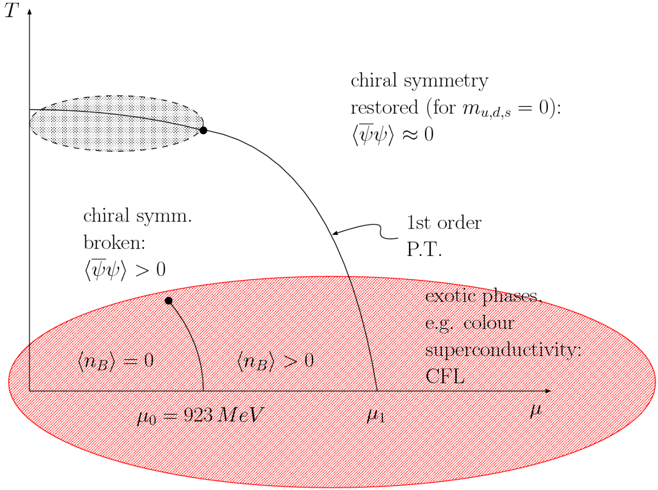



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# Vacuum to nuclear matter: transition at $\mu_0$

- Consider partition function:
  $Z = \sum_{\text{all states } \alpha} \exp\left(-\frac{E_\alpha - \mu N_\alpha}{T}\right)$
- At $T = 0$: sum exponentially dominated by state which minimizes $E_\alpha - \mu N_\alpha$
- At $\mu = 0$: $N = E = 0$ with $n(\mu) = 0$

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- At $T = 0$: sum exponentially dominated by state which minimizes $E_\alpha - \mu N_\alpha$
- At $\mu = 0$: $N = E = 0$ with $n(\mu) = 0$
- $\mu > 0$:
  - $E_\alpha - \mu N_\alpha > 0$: still $N = E = 0$ dominating
  - $E_\alpha - \mu N_\alpha \leq 0$: other states contribute
- Therefore expect phase transition to $n(\mu) > 0$ at

$$\mu_0 := \min_\alpha \left(\frac{E_\alpha}{N_\alpha}\right)$$

- $n(\mu)$ is order parameter for the transition

# Vacuum to nuclear matter: value of $\mu_0$

- Problem: find value of $\mu_0$ for reduced and full QCD
- Reduced QCD:
  - Use $\frac{E}{N} = m_N - \frac{Nm_N - E}{N}$
  - Maximize second term $\epsilon = \frac{Nm_N - E}{N}$ which is binding energy per nucleon
  - Weizsaecker formula, w/o e.-m. interaction and for "infinitely large" nucleus: $\epsilon \approx 16$ MeV
  - Find first-order phase transition to $n_0 \approx 0.16\,\text{fm}^{-3}$ at

$$\mu_0 \approx m_N - 16 \text{ MeV} \approx 923 \text{ MeV}$$



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- Full QCD with Coulomb forces:
  - Infinite nucleus unstable due to Coulomb repulsion
  - Highest binding energy: iron nuclei $\longrightarrow$ adding electrons for neutrality, phase transition to iron solid at $\mu_0 \approx m_N - 8$ Mev $\approx 931$ MeV



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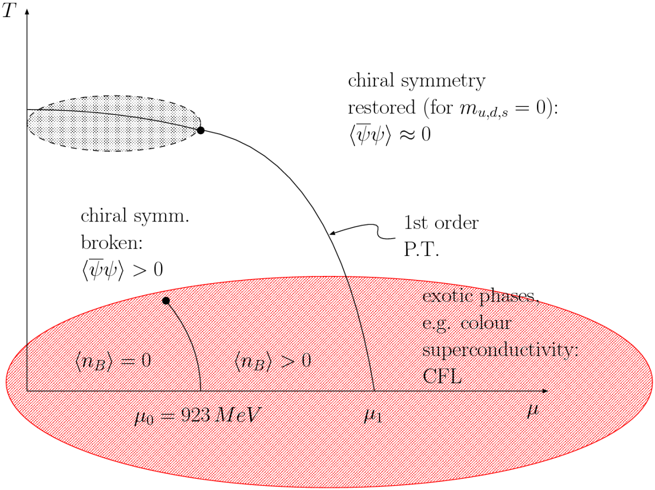




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# High-density phases

- $\mu_0 < \mu < \mu_0 + 200$ MeV: very little known



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# High-density phases

- $\mu_0 < \mu < \mu_0 + 200$ MeV: very little known
- $\mu \gg \mu_0 + 200$ MeV: particles occupy high momentum states due to Fermi statistics
- High momentum $\rightarrow$ asymptotic freedom $\rightarrow$ chiral condensate vanishes
- Restoration of chiral symmetry (exact for massless quarks) is accompanied with phase transition at $\mu = \mu_1$
- Speculation: more phase transitions, exotic phases: colour superconductivity



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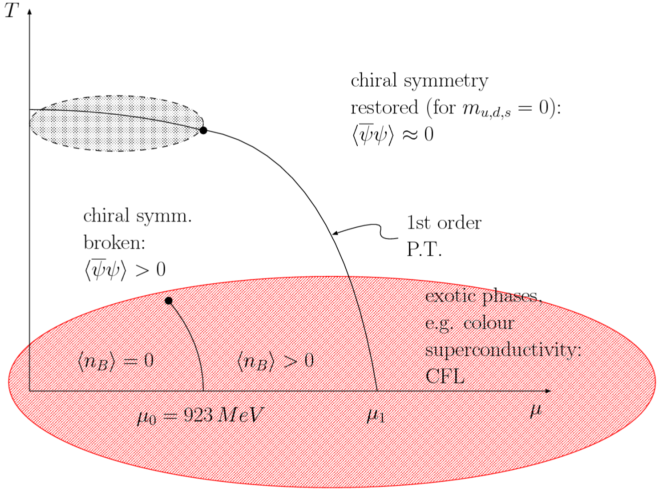




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# The situation for low temperatures — Overview

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- Asymptotic freedom for high momentum states still valid for finite temperatures
- Phase transition at $\mu = \mu_1$ not well understood, therefore little known about low-temperature behaviour
- At $\mu = \mu_0$:
  - $n(\mu) > 0$ for finite $T$ even for $\mu < \mu_0 \rightarrow$ no order parameter
  - But: discontinuities in 1st order phase transitions are assumed to appear as lines of 1st order p.t.s
  - First-order phase transitions should form lines terminated by critical point

# The transition at $\mu = \mu_0$ for low temperatures

- Slope governed by Clausius-Clapeyron: $\frac{dT}{d\mu} = -\frac{\Delta n}{\Delta s}$



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  - Therefore $\Delta s = 0$ at $T = 0 \to$ infinite slope
  - Gas (high $T$) expected to have lower particle density $\to \Delta n < 0$
  - At 1st order transition, system absorbs heat: $\delta Q < 0 \to \Delta s < 0$
  - With $\frac{\Delta n}{\Delta s} > 0$, we find

$$\frac{dT}{d\mu} < 0$$



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- Expect line to terminate in critical point, $T_0 \approx \epsilon \approx 16$ MeV

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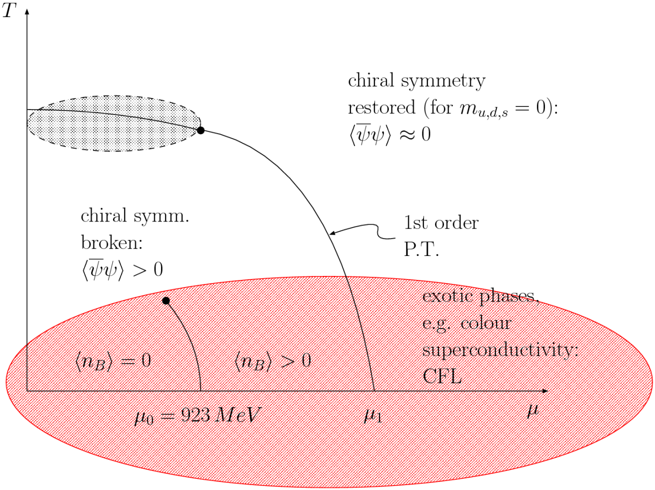





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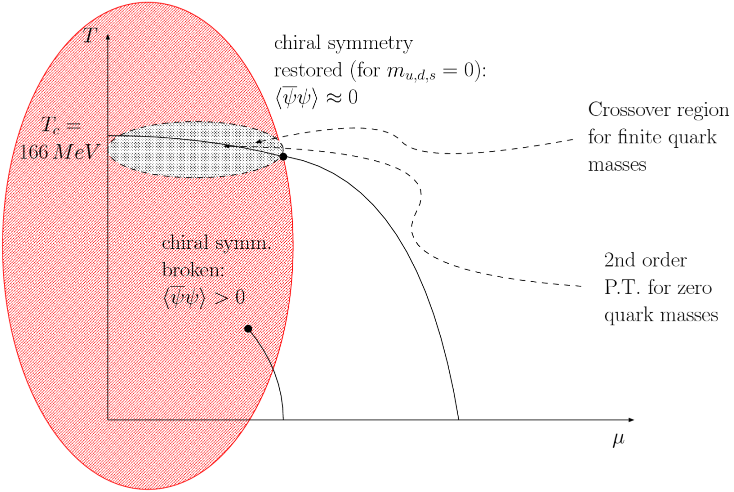

# The QGP

- Raise temperature, keep chem. potential at $\mu = 0$
- Hadronic matter: dominated by pions as lightest mesons
- QGP: high-energy plasma of essentially free quarks and gluons, chiral symmetry restored



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- QGP: high-energy plasma of essentially free quarks and gluons, chiral symmetry restored
- Assumptions
  - Ignore interactions in hadronic phase
  - Only pions formed in hadronic phase
- Find transition: Helmholtz free energy $\Omega = -pV$ minimized
- Pressure is maximized $\rightarrow$ calculate pressure in the two phases

# QGP transition: counting degrees of freedom

For non-interacting fields ($n_f$ number of degrees of freedom):

Bosonic field $3P = \epsilon_B = n_f \frac{\pi^2}{30} T^4$

Fermionic field $3P = \epsilon_F = n_f \frac{7}{8} \frac{\pi^2}{30} T^4$



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For non-interacting fields ($n_f$ number of degrees of freedom):

Bosonic field $3P = \epsilon_B = n_f \frac{\pi^2}{30} T^4$

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$$T_c \approx 150\text{MeV}$$
- First thought to be of first order
- Not seen experimentally!
- Current view:

  Zero quark mass Second order

  Non-zero quark mass No chiral symmetry $\rightarrow$ no symmetry breaking $\rightarrow$ crossover region

The Phases of Quantum Chromodynamics

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# Summary of known transitions

- Two lines of first order phase transitions along $T \approx 0$ and $\mu > 0$
- Second order p.t. (massless quarks) or crossover (finite quark masses) along $\mu \approx 0$ and $T > 0$
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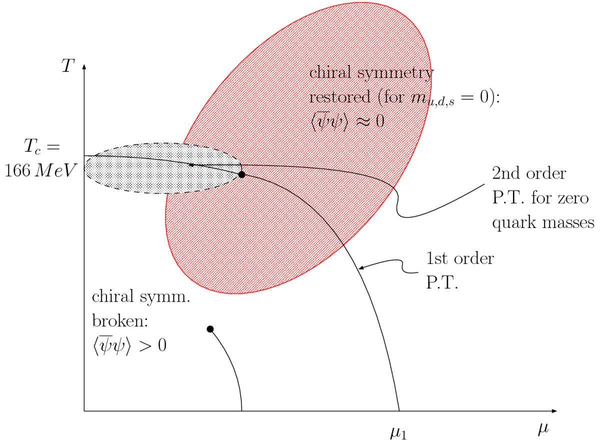
- Chiral symmetry restoration in the region of high temperature and high chem. potential?

# The QCD Phase Diagram — schematically



# Chiral symmetry restoration in the phase diagram

- Massless quarks:
  - Expect **one** line of phase transitions between regions of $\langle Q\overline{Q}\rangle = 0$ and $\langle Q\overline{Q}\rangle \neq 0$
  - Line of 2nd order p.t.s from the QGP transition and of 1st order p.t.s from the $\mu = \mu_1$ transition merge!
  - Meet in a tricritical point

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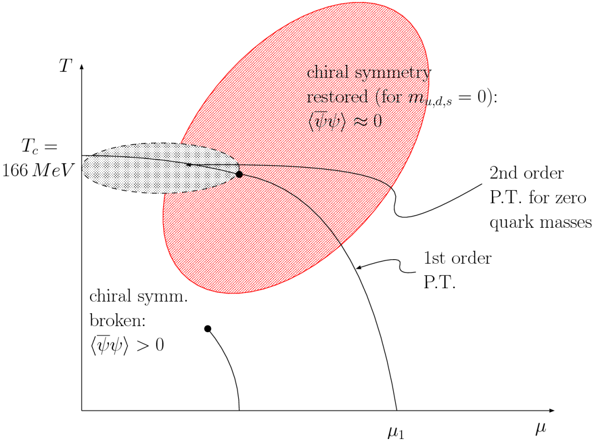
  - Meet in a tricritical point
- Finite quark masses:
  - Chiral symmetry is never exactly restored
  - Line of first order phase transitions from $\mu = \mu_1$ does not terminate in tricritical point

# The QCD Phase Diagram — schematically



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- Densities difficult to produce in earthly laboratories
- Only due to gravitational collapse: neutron star



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# Ordinary superconductivity

- Ordinary solid: ideal, non-interacting Fermi gas approximation becomes valid for small temperatures
- For some materials: at $T_c$, weak attractive force (mediated by ion lattice) leads to formation of condensate of Cooper pairs
- Described by BCS theory for type I superconductivity
- Physical effects:
  - Gap in the excitation spectrum: binding energy of Cooper pairs
  - Zero electrical resistance
  - Meissner-Ochsenfeld effect: expulsion of magnetic fields

# Superconductivity in the QCD case

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- Limit of high density: asymptotic freedom, expect free Fermi gas
- Attractive forces lead to formation of quark pair condensate:
  - Perturbative effects have small contribution
  - Non-perturbative (instanton) effects, proposed in late 1990s, have very large effect
- Colour charge superconductivity and Meissner-Ochsenfeld are not observable
- Energy gap: $\Delta = \frac{E}{2}$, $E$ binding energy
- Estimate: $\Delta \approx 100$ MeV

# Colour flavour locking

- Single-gluon exchange and non-perturbative effect provide attractive force between quarks
- Leads to formation of a condensate (analogous to Cooper pairs; $(\alpha, \beta)$ refer to colour, $(i, j)$ to flavour):

$$\langle \psi_{iL}^{a\alpha}(\vec{p}) \psi_{jL}^{b\beta}(-\vec{p}) \epsilon_{ab} \rangle = -\langle \psi_{iR}^{a\alpha}(\vec{p}) \psi_{jR}^{b\beta}(-\vec{p}) \epsilon_{ab} \rangle \propto \epsilon^{\alpha\beta A} \epsilon_{ijA}$$

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- Locking of flavour and colour indices!
- Breaking of symmetries:

$$SU(3)_c \times SU(3)_L \times SU(3)_R \rightarrow SU(3)_{\text{color,L,R}}$$

- Chiral symmetry is broken, but by another mechanism

# Pseudo-Goldstone bosons

- Spontaneous symmetry breaking:
  - Global symmetry: massless Goldstone boson
  - Local gauge symmetry: gauge bosons acquire mass



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  - Colour symmetry broken $\rightarrow$ gluons acquire mass
  - Chiral symmetry broken: an octet of Goldstone bosons, oscillations of the diquark condensate
- If quark masses are non-zero, pseudo-Goldstone bosons are created with finite, but small, masses
- Can be considered as physical mesons with finite, computable masses: at $\mu = 400$ MeV, $m_{K^{\pm}} \approx 5 \ldots 20$ MeV

# Neutron star cooling

- Neutron star:
  - Star between 1.44x and 3x mass of the sun
  - Density $\approx 10^{12}\text{kg}/\text{cm}^3$ in the core
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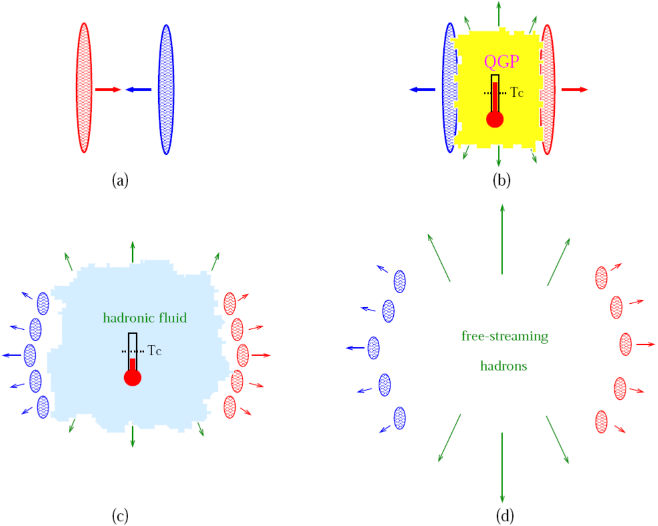
- Takes place via neutrino emission
- Rate of cooling depends on heat capacity
- In CFL: all quarks acquire gap with $\Delta \gg T$, leading to low thermal excitation
- No contribution to specific heat from quarks, specific heat dominated by electrons and pseudo-Goldstone bosons

# RHIC

- Objective: study QGP at high temperatures and low chemical potentials
- Collide heavy nuclei (sulphur, lead, gold) with high energies (A: nucleus weight):
  - Old: SPS @ CERN: CM energy 2A . . . 18 A GeV
  - Current: RHIC @ Brookhaven, NY: 200A GeV
  - Future: LHC (ALICE) @ CERN: 5500A GeV
- Other experiments will at some point attempt to study high-density matter, i.e. CBM/FAIR at GSI, Darmstadt

# Stages of a RHIC collision



*Thank you for your attention*