Quantum Field Theory and Deconfinement Transition

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Introduction

Outline

1. Quantum Field Theory
2. The Wilson Loop and the Static $q\bar{q}$-Potential
3. QFT at Finite Temperature
4. The Wilson Line or Polyakov Loop
5. The Deconfinement Phase Transition
6. Conclusion

Quantum Field Theory

A Few Concepts of QFT

We consider a free scalar field $\phi(x)$

Action $S = \int d^4x\mathcal{L}$

Lagrangian $\mathcal{L}_{\text{scalar}} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}M^2\phi^2$

Fields $\phi(x) = q_{\mathbf{x}}(t)$ become field operators $\Phi(x)$

Correlation Function $G(x, y, \dots) = \langle\Omega|T\left(\Phi(x)\Phi(y)\dots\right)|\Omega\rangle$

Path Integral Representation $G(x, y, \dots) = \frac{\int \mathcal{D}\phi\, \phi(x)\phi(y)\dots e^{iS[\phi]}}{\int \mathcal{D}\phi\, e^{iS[\phi]}}$

Euclidean Space-Time

continuation to imaginary times: $x^0 \to -ix_4$
$\Rightarrow g_{\mu\nu} \longrightarrow \delta_{\mu\nu}$

Coordinates x_μ, $\mu = 1, 2, 3, 4$

Green Functions $\langle \phi(x)\phi(y)\ldots\rangle = \frac{\int \mathcal{D}\phi\,(\phi(x)\phi(y)\ldots)e^{-S_E[\phi]}}{\int \mathcal{D}\phi\, e^{-S_E[\phi]}}$

Euclidean action $S_E[\phi] = \frac{1}{2}\int d^4x\phi(x)\left(-\Box + M^2\right)\phi(x)$

$\Box = \sum_{\mu=1}^4 \partial_\mu\partial_\mu$ is the 4-dimensional Laplacean

Lattice

Introduce a space-time lattice to regularise the path integrals:

Lattice constant a and discrete coordinates $n = (n_1, \ldots, n_4) \in \mathbb{Z}^4$

Substitutions:

$$x_\mu \to n_\mu a \qquad \int d^4x \to a^4 \sum_\mu$$

$$\phi(x) \to \phi(na) \qquad \mathcal{D}\phi \to \prod_n d\phi(na)$$

$$\Box\phi(x) \to \frac{1}{a^2}\hat{\Box}\phi(na) := \frac{1}{a^2}\sum_\mu \left(\phi(na+\hat{\mu}a) + \phi(na-\hat{\mu}a) - 2\phi(na)\right)$$

Correlation Functions on the Lattice

$$\langle \hat{\phi}_n \hat{\phi}_m \dots \rangle = \frac{\int \prod_l d\hat{\phi}_l \, \hat{\phi}_n \hat{\phi}_m \dots e^{-S_E[\hat{\phi}]}}{\int \prod_l d\hat{\phi}_l \, e^{-S_E[\hat{\phi}]}}$$

$$S_E\left[\hat{\phi}\right] = -\frac{1}{2} \sum_{n,\hat{\mu}} \hat{\phi}_n \hat{\phi}_{n+\hat{\mu}} + \frac{1}{2}\left(8 + \hat{M}^2\right) \sum_n \hat{\phi}_n \hat{\phi}_n$$

$$\hat{\phi}_n = a\phi(na)$$

$$\hat{M} = aM$$

$U(1)$ Gauge Invariance

Dirac action for free fermions:
$S_F^{(0)} = \int d^4x\, \overline{\psi}(x)\, (i\gamma^\mu \partial_\mu - M)\, \psi(x)$
Invariant under $\psi(x) \to G\psi(x)$ where $G = e^{i\Lambda} \in U(1)$

Goal: Promote global symmetry to local symmetry $\to$ invariance under local transformation: $\psi(x) \to G(x)\psi(x)$

Solution: covariant derivative $D_\mu = \partial_\mu + ieA_\mu$ with gauge field $A_\mu(x)$

Dynamics for the gauge field: $S_G = -\frac{1}{4}\int d^4x F_{\mu\nu}F^{\mu\nu}$

Gauge Invariance on the Lattice

Local gauge transformation $\psi(n) \to G(n)\psi(n)$

Problem with terms involving fields on two neighbouring lattice sites, $\overline{\psi}(n)\psi(n+\hat{\mu}) \to \overline{\psi}(n)G^{\dagger}(n)G(n+\hat{\mu})\psi(n+\hat{\mu})$

Solution: Insert a link variable: $\overline{\psi}(n)U_{\mu}(n)\psi(n+\hat{\mu})$

Behaviour under gauge transformation:
$U_{\mu}(n) \to G(n)U_{\mu}(n)G^{-1}(n+\hat{\mu})$

Relation to gauge potentials: $U_{\mu}(n) = e^{ieaA_{\mu}(n)}$

Remark: sometimes we will write $U_{n\mu}$ instead of $U_{\mu}(n)$

Link Variables

$U_\mu(n)$ are new fields living on the lattice links

They need their own dynamics

From the structure $F_{\mu\nu}F_{\mu\nu}$, the action involves the smallest loops, or plaquettes in the $\mu-\nu$-plane:

$$U_{n,\mu\nu} := U_\mu(n)U_\nu(n+\hat{\mu})U_\mu^\dagger(n+\hat{\nu})U_\nu^\dagger(n)$$

$$U_{n,\mu\nu} = e^{iea^2 F_{\mu\nu}(n)}$$

$F_{\mu\nu}(n) =$ discrete version of field-strength tensor

Kinetic Term for the Link Variables

For small a: $\frac{1}{e^2}\sum_n \sum_{\mu<\nu}\left[1-\frac{1}{2}\left(U_{n,\mu\nu}+U^\dagger_{n,\mu\nu}\right)\right] \approx$
$\frac{1}{4}\sum_{n,\mu,\nu} a^4 F_{\mu\nu}(n)F_{\mu\nu}(n) \rightarrow \frac{1}{4}\int d^4x F_{\mu\nu}(x)F_{\mu\nu}(x)$

$$S_G[U] = \frac{1}{e^2}\sum_P \left[1-\frac{1}{2}\left(U_P+U_P^\dagger\right)\right]$$

$\sum_P$ describes a sum over all plaquettes

$SU(N)$ Pure Gauge Theory

Link variables $U_\mu(n) \in SU(N)$

Kinetic term in the action:

$$S_G^{(SU(N))}[U] = \beta \sum_P \left[1 - \frac{1}{N} \mathrm{Re}\,\mathrm{Tr}\, U_P\right], \quad \beta = \frac{2N}{g_0^2}$$

Correlation functions: $\langle U_{\mu_1}^{cd}(n_1)\ldots\rangle = \frac{\int \mathcal{D}U\, U_{\mu_1}^{cd}(n_1)\ldots e^{-S_G}}{\int \mathcal{D}U\, e^{-S_G}}$

$\mathcal{D}U$ is a product of Haar measures for $SU(N)$ on each lattice site

The Wilson Loop and the Static $q\overline{q}$-Potential

Procedure

- Want to compute the potential of a pair of static quarks $(M = \infty)$
- Use a gauge invariant trial state
- Compute its euclidean time evolution
- Extract its lowest energy
- For simplicity, we consider first a $U(1)$ theory

Trial State

$$|\phi_{\alpha\beta}(\mathbf{x},\mathbf{y})\rangle = \overline{\Psi}_\alpha(\mathbf{x},0)U(\mathbf{x},0;\mathbf{y},0)\Psi_\beta(\mathbf{y},0)|\Omega\rangle$$

$$U(\mathbf{x},t;\mathbf{y},t) = \exp\left(ie\int_{\mathbf{x}}^{\mathbf{y}} dz^i A_i(\mathbf{z},t)\right)$$

For a non-abelian gauge theory, this last expression would not be so simple.

Evolution

$$G_{\alpha'\beta',\alpha\beta}(\mathbf{x}',\mathbf{y}';\mathbf{x},\mathbf{y};t) =$$
$$\langle\Omega|T\left(\overline{\Psi}_{\beta'}(\mathbf{y}',t)U(\mathbf{y}',t;\mathbf{x}',t)\Psi_{\alpha'}(\mathbf{x}',t)\right.$$
$$\left.\overline{\Psi}_{\alpha}(\mathbf{x},0)U(\mathbf{x},0;\mathbf{y},0)\Psi_{\beta}(\mathbf{y},0)\right)|\Omega\rangle$$

Extraction of the Ground State Energy

Continuation to imaginary times $t \to -iT$
Limit $T \to \infty$

$$G_{\alpha'\beta',\alpha\beta}(\mathbf{x}',\mathbf{y}';\mathbf{x},\mathbf{y};-iT) \to$$
$$\delta^{(3)}(\mathbf{x}-\mathbf{x}')\delta^{(3)}(\mathbf{y}-\mathbf{y}')C_{\alpha'\beta',\alpha\beta}(\mathbf{x},\mathbf{y})e^{-E(R)T}$$

$C_{\alpha'\beta',\alpha\beta}$: overlap with the ground-state
$E(R)$: potential energy for separation $R = |\mathbf{x}-\mathbf{y}|$

Path Integrals

Use path integrals to compute the Green Function

Action

$$\begin{aligned} S &= S_G[A] + S_Q\left[\psi, \overline{\psi}, A\right] \\ S_Q\left[\psi, \overline{\psi}, A\right] &= \int d^4x\, \overline{\psi}(x)\left(i\gamma^\mu D_\mu - M_Q\right)\psi(x). \end{aligned}$$

$\longrightarrow$ no dynamical quarks

$\longrightarrow$ pure Yang-Mills theory

Approximation: static case $\Rightarrow \gamma^\mu D_\mu \rightarrow \gamma^0 D_0$

Integration of the Dirac Fields

1. $U(\mathbf{x},0;\mathbf{y},0) =$
$\exp\left(ie\int_{\mathbf{x}}^{\mathbf{y}} dz_i A_i(\mathbf{z},0)\right)$
2. $U(\mathbf{y},0;\mathbf{y},T) =$
$\exp\left(ie\int_{0}^{T} dz_4 A_4(\mathbf{y},z^4)\right)$
3. $U(\mathbf{y},T;\mathbf{x},T) =$
$\exp\left(ie\int_{\mathbf{y}}^{\mathbf{x}} dz_i A_i(\mathbf{z},T)\right)$
4. $U(\mathbf{x},T;\mathbf{x},0) =$
$\exp\left(ie\int_{T}^{0} dz_4 A_4(\mathbf{x},z^4)\right)$

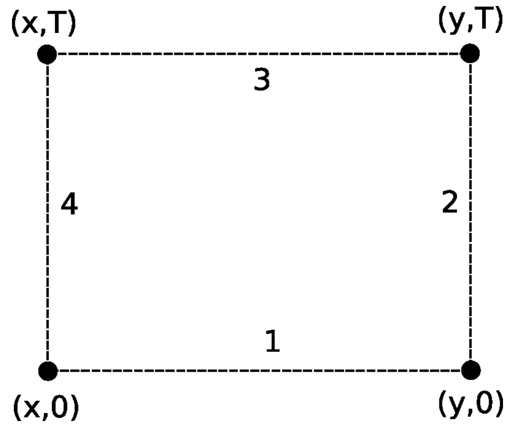

Figure 1: Wilson loop

Wilson Loop

$$W_C[A] = \exp\left(ie \oint_C dz_\mu A_\mu(z) \right)$$

$C \equiv$ Loop with corners at $(\mathbf{x}, 0)$, $(\mathbf{x}, T)$, $(\mathbf{y}, 0)$ and $(\mathbf{y}, T)$

Expectation Value

$$W(R,T) \equiv \langle W_C[A] \rangle_{eucl} = \frac{\int \mathcal{D}A\, W_C[A]\, e^{-S}}{\int \mathcal{D}A\, e^{-S}}$$

$$\begin{aligned} & \lim_{T\to\infty} W(R,T) = F(R) e^{-E(R)T} \\ \Rightarrow \quad & E(R) = -\lim_{T\to\infty} \tfrac{1}{T} \ln\langle W_C[A] \rangle \end{aligned}$$

Lattice Expression

Product of link-variables

$$W_C[U] = \prod_{l \in C} U_l$$

Expectation value:

$$W(\hat{R},\hat{T}) \equiv \langle W_C[U] \rangle = \frac{\int \mathcal{D}U\, W_C[U]\, e^{-S}}{\int \mathcal{D}U\, e^{-S}}$$

$SU(N)$ Theory

In the lattice expression $W_C[U] = \prod_{l \in C} U_l$, simply take the link-variables as elements of $SU(N)$, and make sure the product is path-ordered.

Strong Coupling Limit

Action for $T = 0$ $SU(3)$ gauge theory:

$$S = \beta \sum_P S_P$$
$$\beta = \frac{2N}{g_0^2}, \quad S_P = \frac{1}{N} \mathrm{Re}\,\mathrm{Tr} U_P,$$

Wilson loop expectation value:

$$\langle W_C[U] \rangle = \frac{\int \mathcal{D}U\, W_C[U] \prod_P e^{\beta S_P}}{\int \mathcal{D}U \prod_P e^{\beta S_P}}.$$

Expansion in inverse coupling β:

$$\prod_P e^{\beta S_P} = \prod_P \left[\sum_n \frac{\beta^n}{n!} (S_P)^n \right]$$

Special Case: $SU(3)$

$$\langle W_C[U]\rangle = \frac{\int \mathcal{D}U\, W_C[U] \prod_P e^{\beta S_P}}{\int \mathcal{D}U \prod_P e^{\beta S_P}}.$$

For integrals involving the Haar measure of $SU(3)$:

$$\int dU\, U^{ab} = 0$$

$$\int dU\, U^{ab} \left(U^{\dagger}\right)^{cd} = \frac{1}{3}\delta_{ad}\delta_{bc}$$

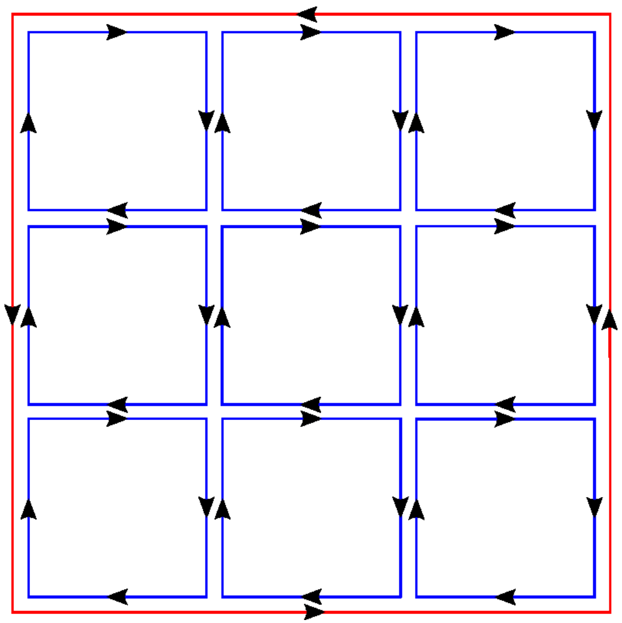

Figure 2: Paved loop

Lowest Order

Lowest order contribution of the numerator: sum up to $n = 1$ for plaquettes inside the loop, and $n = 0$ for the ones outside, i.e. pave the Wilson loop with small loops around each plaquette.
$\longrightarrow$ term proportional to $\beta^{\hat{A}}$, with $\hat{A} = \hat{R}\hat{T}$

Lowest order contribution of the denominator: only 0th order of the exponential.

More exactly, for β small: $\langle W_C[U] \rangle \approx 3 \left(\frac{\beta}{18}\right)^{\hat{R}\hat{T}}$

$\longrightarrow$ linear interquark potential ($\rightarrow$ confinement):

$$\hat{V}(\hat{R}) = -\lim_{\hat{T}\to\infty} \frac{1}{\hat{T}} \ln\langle W_C[U]\rangle \approx -\ln\left(\frac{\beta}{18}\right)\hat{R}$$

QFT at Finite Temperature

Partition Function in QM

$$Z = \mathrm{Tr}\, e^{-\beta H}$$

$$|q\rangle = |q_1, \ldots, q_n\rangle$$

$$Z = \int \prod_{\alpha=1}^{n} dq_\alpha \langle q| e^{-\beta H} |q\rangle.$$

Euclidean time propagator:

$$\langle q'| e^{-H(\tau'-\tau)} |q\rangle \approx \int \mathcal{D}q\mathcal{D}p\, e^{ip_\alpha^{(l)}\left(q_\alpha^{(l+1)} - q_\alpha^{(l)}\right)} e^{-\epsilon H(q^{(l)}, p^{(l)})}$$

$\mathcal{D}q\mathcal{D}p = \prod_{\beta=1}^{n} \prod_{l=1}^{N-1} dq_\beta^{(l)} \prod_{l=0}^{N-1} \frac{dp_\beta^{(l)}}{2\pi}$, $q^{(0)} = q$ and $q^{(N)} = q'$

Path Integral Representation of Z

Replace $\tau' - \tau \to \beta$

$$Z = \lim_{\substack{N\to\infty \\ \epsilon\to 0 \\ N\epsilon=\beta}} \int \mathcal{D}q\mathcal{D}p \, e^{i\phi[q,p]} e^{-\sum_{l=0}^{N-1} \epsilon H\left(q^{(l)},p^{(l)}\right)} \Big|_{q^{(N)}=q^{(0)}}$$

where $\mathcal{D}q\mathcal{D}p = \prod_{l=0}^{N-1} \prod_\alpha \frac{dq_\alpha^{(l)} dp_\alpha^{(l)}}{2\pi}$

and $\phi[q,p] = \sum_{l=0}^{N-1} \sum_\alpha p_\alpha^{(l)} \left(q_\alpha^{(l+1)} - q_\alpha^{(l)}\right)$

(formal) continuum version:

$$Z = \int_{\text{per}} \mathcal{D}q \int \mathcal{D}p \, e^{-\int_0^\beta d\tau \left[\sum_\alpha i p_\alpha(\tau)\dot{q}_\alpha(\tau) - H(q(\tau),p(\tau))\right]}$$

Finite Temperature QFT

Introducing a non-zero temperature is equivalent to introducing periodic boundary conditions on the time-direction, with period equal to inverse temperature β.

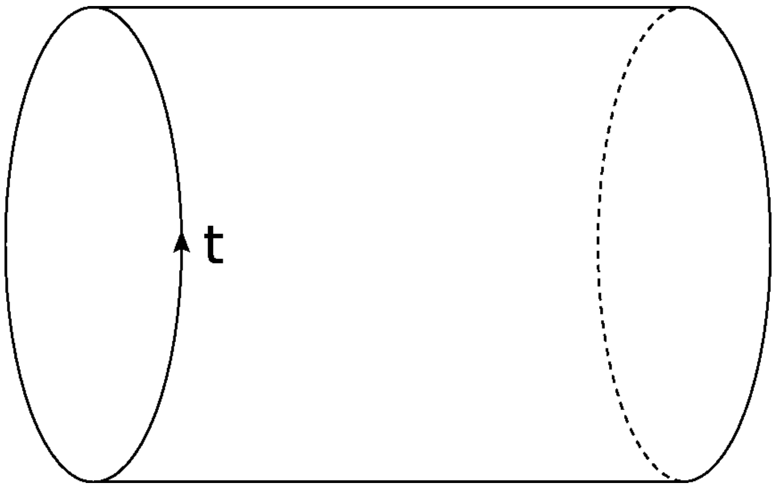

Figure 3: Compactified lattice

Example: Scalar Bosons

Lagrangian of the free scalar theory

$$\begin{aligned} L_E[\phi, \dot{\phi}] &= \int d^3x \mathcal{L}_E(\phi, \partial_\mu \phi), \\ \mathcal{L}_E &= \tfrac{1}{2}\partial_\mu \phi \partial_\mu \phi + \tfrac{1}{2} M^2 \phi^2, \end{aligned}$$

Partition function

$$Z_0 = \mathcal{N} \int_{\text{per}} \mathcal{D}\phi \, e^{-\int_0^\beta d\tau \int d^3x \mathcal{L}_E\left[\phi, \dot{\phi}\right]}.$$

$SU(N)$ Theory at Finite Temperature

Partition function

$$Z = \int_{\text{per}} \mathcal{D}U \, e^{S^{(\beta,\mu)}[U]}$$

Action

$$S = \sum_n \mathrm{Re} \left\{ \beta_t \sum_i \mathrm{Tr}\left(U_{n,i4}\right) + \beta_s \sum_{i<j} \mathrm{Tr}\left(U_{n,ij}\right) \right\}$$

β_t and β_s are independent couplings for "timelike" and "spacelike" plaquettes

The Wilson Line or Polyakov Loop

Non-Trivial Loops

Space-time periodic along time direction $\Rightarrow$ Non-Trivial Loops

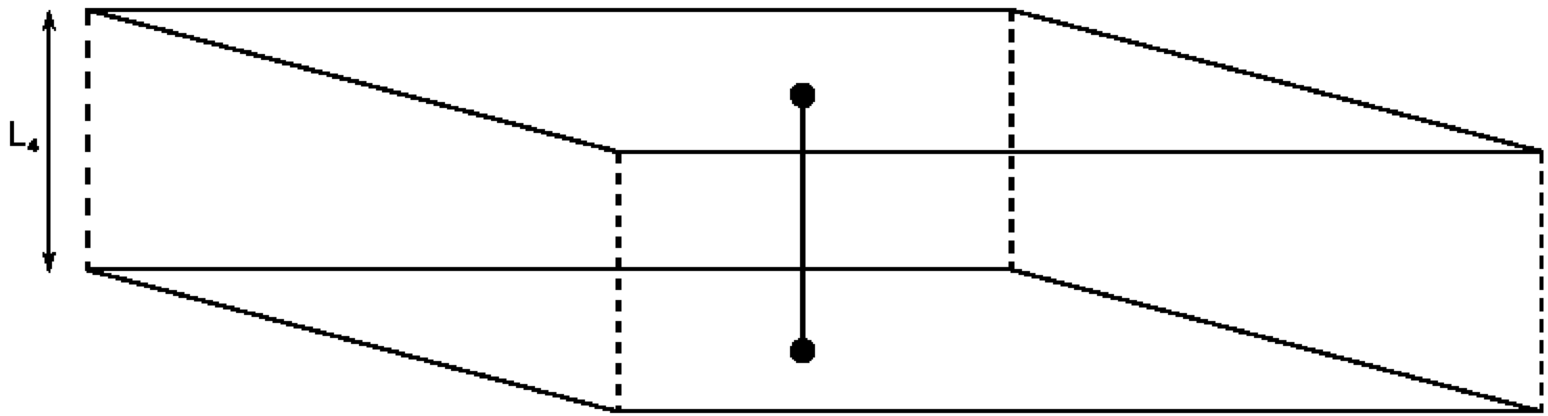

Figure 4: Non-trivial loop

The Wilson Line

Wilson-Line at some spatial location $\mathbf{x}$

$$L(\mathbf{x}) = \mathrm{Tr} \prod_{n=1}^{\hat{\beta}} U_4(\mathbf{x} + n\hat{t})$$

$\longrightarrow$ gauge-invariant since closed

Expectation values and correlation functions $\rightarrow$ Free energy of single quarks and pairs

$$\begin{aligned} e^{-F_q/T} &= \langle L(\mathbf{x}) \rangle \\ e^{-F_{q\overline{q}}(\mathbf{x}-\mathbf{y})/T} &= \langle L(\mathbf{x}) L^\dagger(\mathbf{y}) \rangle \end{aligned}$$

where $\langle \alpha\, [U] \rangle = \dfrac{\int \mathcal{D}U\ \alpha[U]\ e^{S^{(\beta,\mu)}[U]}}{\int \mathcal{D}U\ e^{S^{(\beta,\mu)}[U]}}$

Confinement

Confinement
$\Leftrightarrow F_{q\overline{q}} = \infty$ for $R \to \infty$
$\Leftrightarrow \lim_{|\mathbf{x}-\mathbf{y}|\to\infty} \langle L(\mathbf{x}) L^\dagger(\mathbf{y})\rangle = 0$

Provided clustering holds, i.e.
$\lim_{|\mathbf{x}-\mathbf{y}|\to\infty} \langle L(\mathbf{x}) L^\dagger(\mathbf{y})\rangle = |\langle L\rangle|^2$, then
Confinement $\Leftrightarrow \langle L(\mathbf{x})\rangle = 0$:
Inserting a single quark requires infinite energy

The Deconfinement Phase Transition

The Center of a Group

The action $S = \sum_n \mathrm{Re}\left\{\beta_t \sum_i \mathrm{Tr}\,(U_{n,4i}) + \beta_s \sum_{i<j} \mathrm{Tr}\,(U_{n,ij})\right\}$ has not only gauge symmetry, but also center symmetry.

$\mathcal{C} = \left\{z \in G | zgz^{-1} = g \; \forall g \in G\right\}$ is the center of G

For $G = SU(N)$, $\mathcal{C} = Z_N = \left\{e^{\frac{2\pi i l}{N}} \mathbb{I}_N | l = 0, 1, \ldots, N-1\right\}$

Center Symmetry Transformation

For a fixed n_4, multiply all time-like oriented link-variables of the corresponding space-slab by an element of the center:

$$U_4(\mathbf{n}, n_4) \to z\, U_4(\mathbf{n}, n_4), \quad z \in \mathcal{C}$$

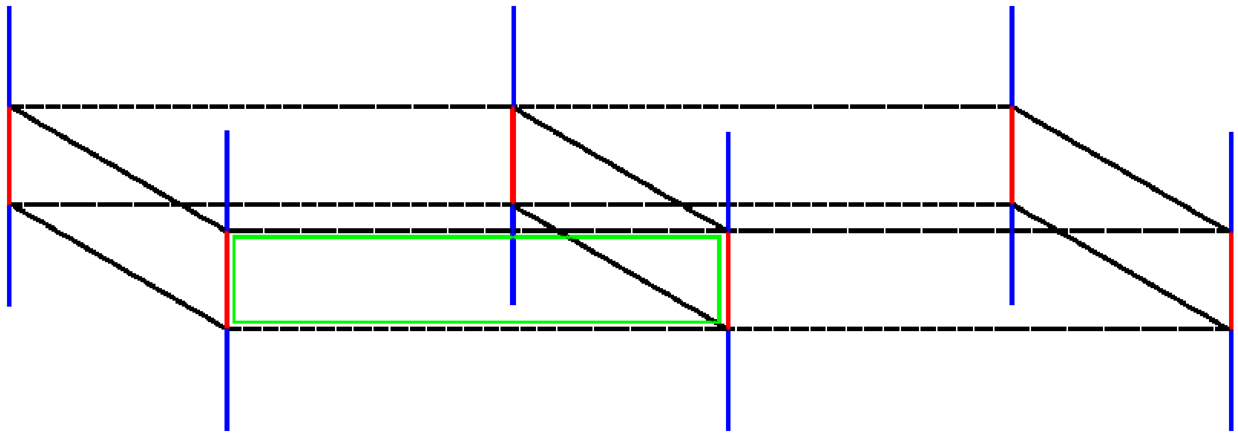

Figure 5: Slab of constant n_4

The Center Symmetry (2)

Under center transformations, only time-like plaquettes in the fixed n_4 slab are involved:

$$U_{n,4i} = U_4(n)U_i(n+\hat{4})U_4^\dagger(n+\hat{i})U_i^\dagger(n) \rightarrow$$
$$z\, U_4(n)U_i(n+\hat{4})z^\dagger U_4^\dagger(n+\hat{i})U_i^\dagger(n) = U_4(n)U_i(n+\hat{4})U_4^\dagger(n+\hat{i})U_i^\dagger(n)$$

$\Rightarrow S$ is kept invariant

$L(x)$ is not:

$$L(\mathbf{x}) \rightarrow zL(\mathbf{x})$$

Relation to Confinement

If the ground state of the quantum system respects the symmetry of the classical action, the same number of configurations will yield the values

$$L_l = e^{2\pi i l/N} L, \quad (l = 0, 1, \ldots, N-1)$$

for the Wilson line.
But $\sum_{l=0}^{N-1} e^{2\pi i l/N} = 0$, hence

$$\langle L \rangle = 0$$

$\longrightarrow$ confinement

Conversely, $\langle L \rangle \neq 0 \;\Rightarrow$ center symmetry is broken

Deconfinement transition $\longleftrightarrow$ Breakdown of center symmetry, phases of the Polyakov loop cluster around one of the Z_N roots

High Temperatures

$$S = \sum_n \mathrm{Re}\left\{\beta_t \sum_i \mathrm{Tr}\left(U_{n,4i}\right) + \beta_s \sum_{i<j} \mathrm{Tr}\left(U_{n,ij}\right)\right\}$$

For high temperatures, with $\beta_t \sim T$, main contribution comes from configurations with $U_{n,4i} = 1$:

$$U_{\mathbf{n}i} = U_{\mathbf{n}4} U_{\mathbf{n}+\hat{t},i} U^{\dagger}_{\mathbf{n}+\hat{i},4}$$

Periodicity requires

$$U_{\mathbf{n}i} = U_{\mathbf{n}+N_t\hat{t},i}$$

These conditions imply for $\Omega_{\mathbf{x}} \equiv \prod_{n=1}^{\hat{\beta}} U_4(\mathbf{x}+n\hat{t})$ ($L(\mathbf{x}) = \mathrm{Tr}\Omega_{\mathbf{x}}$):

$$\Omega_{\mathbf{x}} = U_{\mathbf{x}i}\Omega_{\mathbf{x}+\hat{i}}U^{\dagger}_{\mathbf{x}i}$$

High Temperatures (2)

For most of the configurations, this requires

$$\Omega_{\mathbf{x}} \equiv z, \quad z \in \mathcal{C} \text{ constant}$$

This implies that $\langle L \rangle \neq 0$, the center symmetry is broken and the system is in a deconfining phase.

High Temperatures (3)

Alternative argument:

$\longrightarrow$ asymptotically free theory

$\longrightarrow$ high $T \Rightarrow$ weak coupling

$\longrightarrow$ quarks are deconfined

Phase Transition

If $T = 0$ is confining, then there exists a phase transition separating the low-temperature confining phase from the high-temperature deconfining phase

Order parameter: Wilson line $\langle L(x) \rangle$

Two Phases

Confining Phase	Deconfining Phase
$T \sim 0$	$T \to \infty$
Symmetry realised	Symmetry broken
$\langle L \rangle = 0$	$\langle L \rangle \neq 0$
$F_q = \infty$	$F_q < \infty$

Svetitsky-Yaffe Conjecture

Assume transition is 2nd order

Correlation function $\Gamma(\mathbf{x}-\mathbf{y}) \equiv \langle L(\mathbf{x}) L(\mathbf{y})^\dagger \rangle$

$\lim_{|\mathbf{x}-\mathbf{y}|\to\infty} \Gamma(\mathbf{x}-\mathbf{y}) = |\langle L \rangle|^2$

If $R \gg \beta$, integrating out the spatial gauge fields of the original $(d+1)$ field theory, we get an effective d-dimensional system of Polyakov loops interacting via H_{eff}

Conjecture: Assume finite (short) range interaction (range $= \rho$), then this effective theory belongs to the universality class of d-dimensional spin-system of the center symmetry.
$\longrightarrow$ predictions about the critical behaviour

Predictions

- (3+1)-dimensional gauge theories:
  - $SU(2) \to$ Center $Z_2 \to$ 3-dimensional Ising model
  - $SU(3) \to Z_3$, but the transition is 1st order
- (2+1)-dimensional gauge theories:
  - $SU(2) \to$ 2d Ising model
  - $SU(3) \to$ 2d, $q = 3$ Potts model
  - $U(1) \to$ 2d XY model, Berezinsky-Kosterlitz-Thouless transition

Some Numerical Results for $SU(3)$

Locate the Transition

- On infinite lattices: $\langle L \rangle$ is an order parameter
- On finite lattices: tunneling $\Rightarrow \langle L \rangle \equiv 0$

$\longrightarrow$ we look at $\langle |L| \rangle$:

- In the confining phase, $\langle |L| \rangle$ is small
- In the deconfining phase, $\langle |L| \rangle$ is large

Alternatively, test the Z_3 symmetry directly by looking at the distribution of real and imaginary parts of Polyakov loops:

- In the Z_3 symmetric phase, configurations related by Z_3 symmetry operations occur with equal probability.
- In the Z_3-broken phase the system will spend a substantial simulation time in one of the three vacua, before tunnelling between the vacua will restore the Z_3 symmetry.

Confined Phase

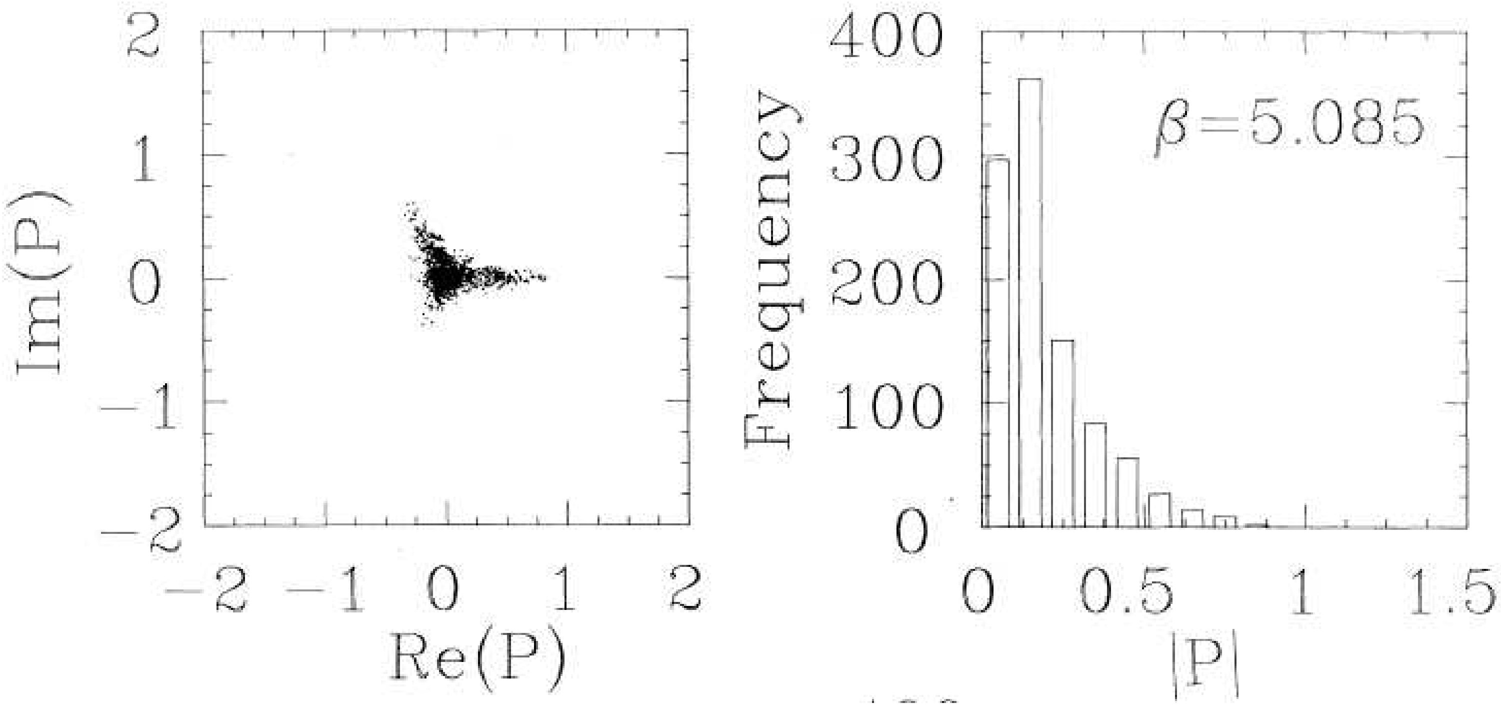

Figure 6: The center symmetry is realised

Deconfined Phase

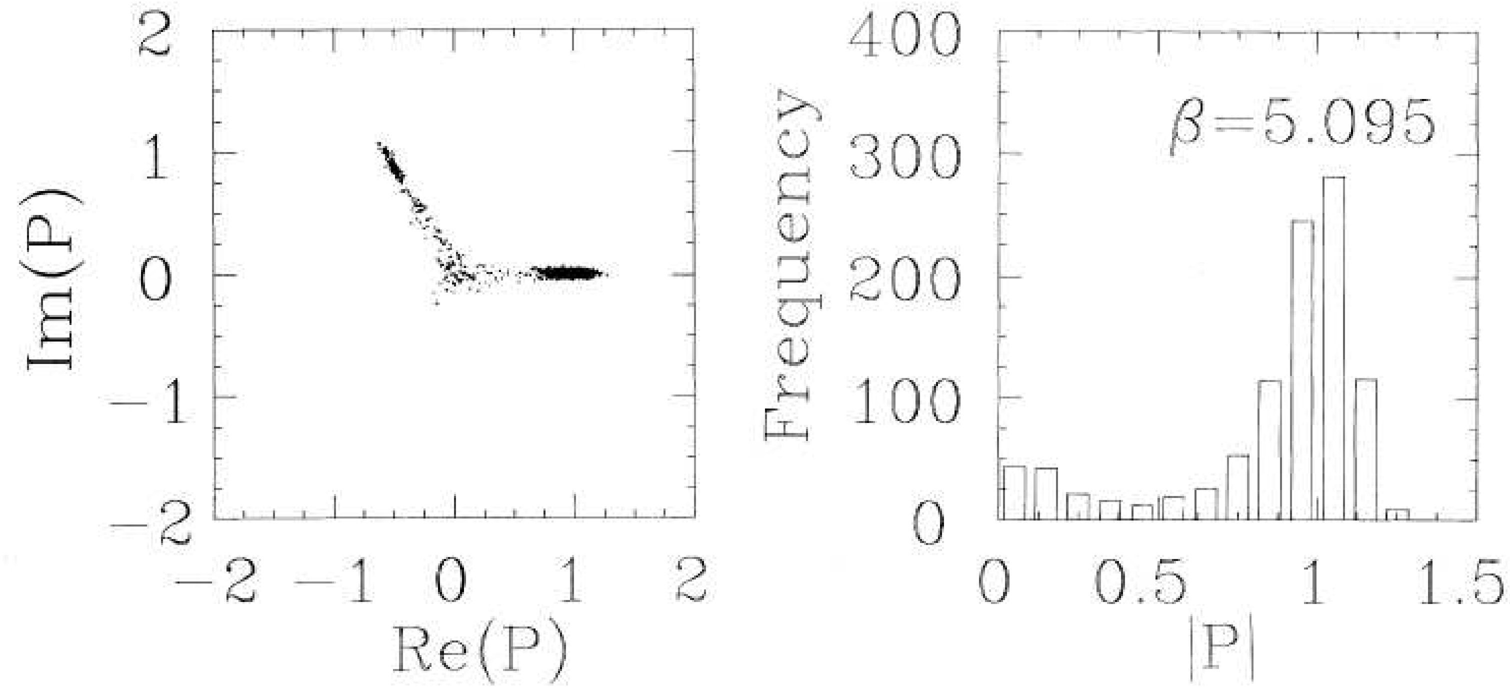

Figure 7: The center symmetry is broken

Phase Transition

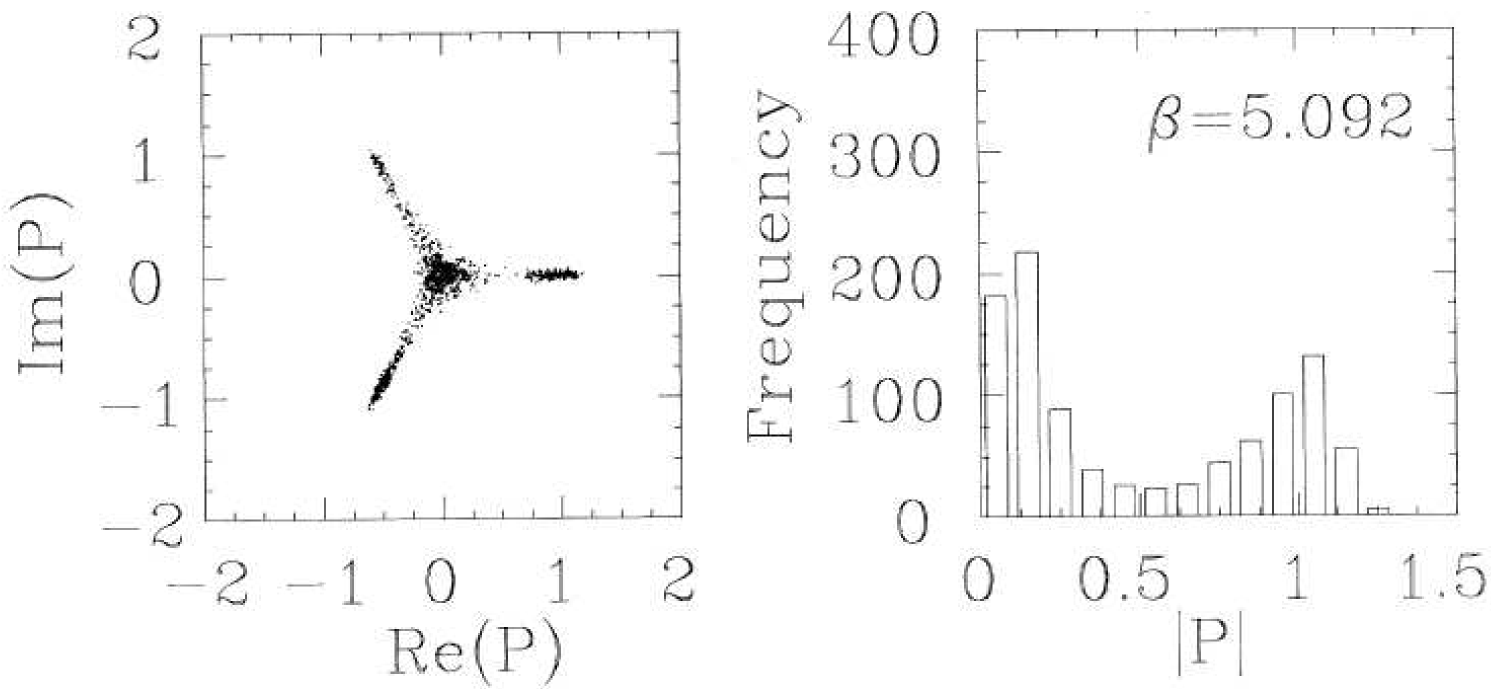

Figure 8: The two phases coexist

QCD

For QCD values of the coupling: $T_{\mathrm{crit}} \sim 10^{12}\,\mathrm{K} \rightarrow \sim 270\mathrm{MeV}$

This was reached $\sim 10^{-6}\,\mathrm{s}$ after the Big Bang

Thank you

and have a nice day!