Berry Phase 1

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The dynamics of a quantum system is determined by the time-dependent Schrödinger equation $H(R)\psi(t) = id\psi(t)/dt$. If the physical system is not isolated from its environment, the Hamiltonian operator $H(R)$ depends on external or environmental factors R. A changing environment is described by time-dependent parameters $R(t)$. If the external factors change periodically, the quantum system performs a cyclic evolution. This means that after each turn of such an evolution a state vector evolves into a vector which agress with the inital vector only up to a phase factor. In addition to the usual dynamical phase, this phase factor contains a purely geometric part which does not depend on the duration of the evolution. In the quantum adiabatic approximation, i.e. when the environment and therefore the Hamiltonian changes slowly compared to the rate of change of the dynamic itself, the geometric phase factor is called a Berry phase $\gamma_n(C)$.

For a quantum system in a classical environment, the Hamiltonian $H(R)$ lives on a parameter manifold M, where any point $R \in M$ describes a different environment. The geometric phase can be expressed in terms of a vector potential one-form or a curvature two-form acting on the manifold. Such a reformulation of the Berry phase leads to a deeper insight of the mathematical structures and simplifies calculations in many instances.

As an example, we shall study the case of a closed circuit C on a three dimensional flat parameter manifold which lies near a degeneracy of the Hamiltonian. For such a configuration it is found that the Berry phase is directly connected to the solid angle which is subtended by the curve, i.e. $e^{i\gamma_n(C)} = e^{-i\frac{\Omega(C)}{2}}$. A similar relation between the Berry phase and the area enclosed by the traversed circuit C is also found for a Hamiltonian which describes spins precessing adiabatically in an external magnetic field. Furthermore, it is shown that the Aharonov-Bohm effect can be interpreted as a geometrical phase factor. And finally we present a neutron interferometer experiment, which allows to directly measure the Berry phase: A z-polarized neutron beam is splitted into two separate beams, which are guided through two identical spin flippers. These flippers flip both beams by π. As there is an angle between the flippers a geometric phase emerges, which can be detected by interference of the two beams.