Spin and Charge Density Waves

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In certain materials with a highly anisotropic band structure and metallic behaviour instabilities leading to a spin or charge density redistribution can occur. It is the goal of this talk to treat these density wave formations within a linear response theory and a mean field approximation.

First of all, the Lindhard response function, describing the response of a density to a potential, is presented in various dimensions within the nearly free electron approximation and is shown to be, in the one dimensional case, divergent at momentum transfer $2k_F$ (k_F is the Fermi-wavevector). A divergent response function means that we can have spontaneous self-ordering. The concept of nesting illustrates the gradual removement of the instability with increasing dimension. This is not necessarily the case if the nearly free electron approximation does not apply.

In the following discussion we thus mainly regard materials with a quasi-one dimensional structure, as for example a compound of linear chains, as being one class of materials developing density waves.

We treat the charge density wave starting from the Fröhlich Hamiltonian, where it is at first shown that the electron-phonon interaction leads to a strongly renormalized phonon spectrum with a kink at $q = 2k_F$, usually referred to as the Kohn anomaly. In the one dimensional case there is even a tempereature, the mean field transition temperature, below which the renormalized phonon frequency is zero and the system undergoes a phase transition (the Peierls transition) and develops a static lattice distortion.

We thus restrict ourselves to the strictly one dimensional case, impose a mean field approximation on the phononic operators and diagonalize the Hamiltonian with the help of quasi-particles. We find that their excitation spectrum develops a gap at the Fermi surface and that the material is therefore turned into an insulator.

The charge density is periodically modified, where the amplitude and the phase, as the size of the single particle gap, are determined by the complex order parameter and the period is given by the Fermi vector. In the general case, where the density modulation is incommensurate with the lattice, translational symmetry is broken.

The value of the order parameter is usually found by solving a self consistency equation. In this case it is done by minimizing the energy. It will also be shown that the single particle gap follows the same temperature dependence as in BCS theory.

Finally, a few experimental evidences will be presented to show that the mean field approximation yields qualitatively good results, but generally leads to transition temperatures which are too high.

The same is then done for the spin density wave within the Hubbard model, where we again find a gap in the spectrum and therefore a metal-insulator transition, a periodic spin density redistribution and that, in general, translational symmetry is broken.

Because we have seen the limits of a mean field approximation concerning quantitative results, a short overview over the effects of fluctuations of the order parameter is given and it is shown that there is no long range order in the strictly one dimensional case. One can then show that in the case of a compound of linear chains, interchain reactions like Coulomb interaction and interchain tunneling lead to a transition temperature,being lower than the mean field transition temperature, below which the correlation length diverges and a three dimensional long range ordered ground state develops.