One-dimensional systems 2

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This talk is about $S = \frac{1}{2}$ spin chains. We will align spins on a periodic one-dimensional lattice and let them interact, look what the ground state is and decide if there is order or not. We also will look at excitations and their energies and study how they affect the ordering of the spins.

We start with the Heisenberg model for spin interaction, showing how it arises from the Hubbard model in the limit of strong interaction at half-filling. Next we describe one dimensional spin chains by making use of the anisotropic version of the Heisenberg model: the XXZ-model. We introduce the notion of *domain walls* and show how in the classical limit (Ising model) the long range order at zero temperature is destroyed by the excitation of domain walls. In the general case we discuss the difficulties which arise when considering the classical ground states and, in the quasi-Ising limit, we see how the excitation of domain walls gives rise to continuous excitation spectrum.

In the second part we give a method which allows to see the spin system and its excitations in another, more fundamental, way. The *kink* operator, a non-local spin operator, is defined and its geometrical meaning explained. Next the Jordan-Wigner transformation is introduced. With its help we can transform the spin system in a system of spinless fermions. Transforming the XXZ-hamiltonian leads to a system wich is nearly identical with the Hubbard model: it has term describing the hopping of the fermions and a term responsible for an on-site interaction between the fermions. In the XY-limit these fermions are free, and we explicitly solve this system. We calculate the energy dispersion relation and see that the excitation of the Jordan-Wigner-fermions leads to a gapless excitation spectrum.