Coherent State Path Integrals, Berry's Phase

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This talk is about geometric phases. A well-known example of a geometric phase is the Aharonov-Bohm-term $\frac{e}{\hbar}\vec{A}\cdot d\vec{x}$.

A coherent state path integral for a spin S will be derived. For $\vec{n} \in S^2$, denote by $|\vec{n}\rangle$ the state vector of a spin pointing in direction $\vec{n}$. If H is the Hamiltonian, and $[t_0, t_1] \ni t \mapsto \vec{n}(t)$ the parametrization of a closed curve c on the sphere, then the corresponding action is given by

$$\int_{t_0}^{t_1} dt\, i\langle\vec{n}(t)|\frac{d}{dt}|\vec{n}(t)\rangle - \int_{t_0}^{t_1} dt\, \langle\vec{n}(t)|H|\vec{n}(t)\rangle. \tag{1}$$

The first term can also be written as $\int_c i\langle\vec{n}|d\vec{n}\rangle$. This is Berry's Phase. It has a simple geometric interpretation:

$$\int_c i\langle\vec{n}|d\vec{n}\rangle = S\cdot(\text{solid angle enclosed by } c), \tag{2}$$

Actually, there is a $U(1)$-gauge arbitrariness in defining $|\vec{n}\rangle$. However, the Berry phase along a closed path is invariant.

As an application, an experiment with so called Single-Molecule-Magnets (SMM's) will be discussed, where Quantum Tunneling of the Magnetization (QTM) can be observed. It is found experimentally that tunneling is qualitatively different for half-integral and integral spin, respectively. This will be explained using (2).

The Berry phase in the path integral for a spin is just an example. More in general, we may consider a path $[t_0, t_1] \ni t \mapsto |\psi(t)\rangle$ in any Hilbert space, where $|\psi(t)\rangle$ is normalized for all t. The action is given by simply replacing $|\vec{n}(t)\rangle$ with $|\psi(t)\rangle$ in (1). Then $\int i\langle\psi|d\psi\rangle$ is called Berry's phase. It is a geometric phase: it depends on the path traversed in parameter space, but *not* on how fast one goes through that path (i.e. the parametrization).

Historically, Berry's Phase was first introduced when considering quantum mechanical systems which are subject to adiabatic changes of the environment. Gauge arguments will be used to derive the Berry Phase in this context. A formal analogy with electromagnetism will be established. In this picture, degeneracies in parameter space play the role of magnetic monopoles, and their quantization is a consequence of the famous argument given by Dirac.

Finally, the Integer Quantum Hall Effect will be discussed. Experimentally it is observed that the conductance of a two-dimensional electron gas is quantized, under suitable conditions, in units of e^2/h. It will turn out that this quantization can be interpreted in terms of topological invariants (Chern-integers). While there will be some simplifications (one particle living in one band), the geometric origin of the result will be emphasized throughout the calculation.