# Path Integral 3

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In 1935, Schrödinger attempted to demonstrate the limitations of quantum mechanics using a thought experiment in which a cat is put in a quantum superposition of alive and dead states. During the past few years, experimentalist groups have shown (directly of indirectly) that coherent superposition of quantum states can even occur on macroscopic scales, provided the system is sufficiently decoupled from its environment. One of the best systems that demonstrates this macroscopic quantum behavior is a superconductor. I will describe two experiments in which superconductor were used. The first one only shows indirectly a MQC (Macroscopic Quantum Coherence) using the energy splitting of the superposition of two states, while the other one demonstrates this behavior by means of Rabi oscillations induced by a monochromatic electromagnetic field. In the last experiment, a lower bound for the decoherence rate was measured: $\tau_{\text{decoh}} \geq 4.9\mu\text{s}$

Next I shall explain the theoretical foundation of coherence by introducing the real-time path integral formalism, which will lead us to the Feynman-Vernon functional. This formalism describes non-equilibrium time-dependent phenomena. In the first part, I study the evolution of the damped system under the assumption that it was prepared initially in a product state of the system-plus-reservoir complex:

$$\hat{W}(0) = \hat{\rho}(0) \otimes \hat{W}_R(0),$$

where $\hat{W}$ is the density matrix of the system-plus-reservoir, $\hat{\rho}$ the density matrix of the system and finally $\hat{W}_R$ the density matrix of the reservoir.

The result of this calculation is

$$\rho(q_f, q_{f'}; t) = \int dq_i dq_i' J_{FV}(q_f, q_{f'}, t; dq_i, dq_i', 0)\rho(dq_i, dq_i'; 0).$$

The indices $i$ and $f$ denote the initial resp. the final state. $J_{FV}$ is the propagating function describing the time evolution of the reduced density matrix:

$$J_{FV}(q_f, q_{f'}, t; dq_i, dq_i', 0) = \int \mathcal{D}q\mathcal{D}q' \exp\left\{\frac{i}{\hbar}\Sigma[q, q'] - \Phi_{FV}[q, q']\right\}.$$

$\Sigma[q, q']$ is the reversible action of the system and $\Phi_{FV}[q, q']$ is the influence weight functional.

The result of this calculation will explain us how decoherence and energy relaxation work. We will see that energy relaxation is due to damping. Next I make a concrete example in order to see why dephasing is much faster than energy relaxation. In other words, we will understand why our macroscopic world behaves "classically".

I will also study a two-state dynamics which describes to a good approximation level the behavior of a SQUID (Superconductor QUantum Interference Device). The dynamics of the SQUID can be described in terms of a one-dimensional double-well potential which can be modified by means of magnetic fields. This is known as the spin-boson model: the environment is described by an oscillator bath coupled to one component of the spin. Two different time scales characterize the evolution. On a first, decoherence time scale $\tau_{\text{decoh}}$ , the off-diagonal elements of the density matrix decay to zero. On the second, relaxation time scale $\tau_{\text{relax}}$, the diagonal entries tend to their thermal equilibrium values.

The decoherence time scale and the relaxation time scale are evaluated in a path-integral technique using the Feynman-Vernon formalism. I shall outline how to compute these two scales using the Non-Interacting Blip Approximation (NIBA). NIBA was introduced by Leggett in 1987 in his famous paper on the dynamics of the dissipative two-state system.