Mean Field I - Superconductivity

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In a wide range of types of material the ability to pass an electrical current without any measurable resistance has been found. This phenomenon called superconductivity, first discovered by H.K. Onnes in 1911, occurs suddenly when the material is cooled below a critical temperature T_c. Although the vanishing of the resistance is perhaps the most spectacular and most famous property in a superconductor, there are other effects like the discontinuity in the specific heat at T_c, the Meissner effect or the isotope effect.
The aim of this talk is to present a microscopic theory of superconductivity explaining all of these phenomena which was introduced by Bardeen, Cooper and Schrieffer (and is therefore refered to as BCS-Theory). For simplicity, we shall concentrate mainly on so-called type I singlet superconductors like tin or aluminum.
First, we will see, that the Fermi sea of electrons is unstable against the formation of bound pairs if there exists a no matter how small weak attraction between electrons. This means, that there is a state of negative energy with respect to the Fermi surface made up entirely of electrons with $k > k_F$.
Starting from the Fröhlich Hamiltonian

$$\sum_k \mathcal{E}_k c_k^\dagger c_k + \sum_q \hbar\omega_q a_q^\dagger a_q + \sum_{k,k'} M_{kk'}(a_{-q}^\dagger + a_q) c_k^\dagger c_{k'}$$

which describes the interaction between electrons and phonons we will then derive the BCS Hamiltonian and see, that there is indeed an attractive interaction between electrons in a crystal lattice. In contrary to the original BCS paper, we will not make a variational approach but try to diagonalize our hamiltonian with first applying a mean field approximation and subsequently a Bogoliubov-Valatin transformation (i.e. we introduce quasiparticles). In so doing we will find a ground state wave function, the ground state energy (which is indeed lower then in the normal, non superconducting state). Furthermore, we observe an energy gap at the Fermi surface in the spectrum of the quasiparticles.
Next, it is presented how the BCS theory provides results for the basic phenomena that are in good agreement with the experimentally found, like the temperature dependence of the energy gap, the isotope effect and the specific heat.
Finally, we calculate the current density **j** in a superconductor when applying a weak magnetic field to first order in the applied field. We therefore consider a magnetic field defined by a vector potential $A = A_q e^{iqr}$ and examine the limit when q becomes very small. Thus, we see, that for $T = 0$ a Meissner effect can be predicted while for $T > T_c$ there is none.