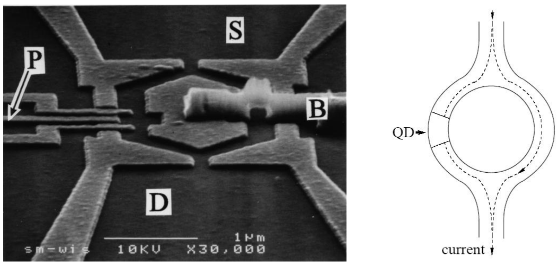
Scattering matrix theory 2

Harry Figi

The transport properties of electronic devices are characterized on the basis of conductance measurements. In mesoscopic devices (a few micrometer) the wave nature of the electrons plays an important role, because the phase of an electron's wave function changes as it passes through such a small device (for example quantum dots) and therefore phase measurements are required to characterize the transport properties fully. The origin of many phenomena by studying electron transport on mesoscopic length scales is the quantum mechanical phase coherence of the electronic wave functions. The degree of coherence can be measured by the phase coherence length, which is the typical length on which electrons travel without loosing their phase coherence. In mesoscopic systems, both the phase coherence length and the elastic mean free path of the electrons exceed the entire sample size. Under these conditions, transport through the system is ballistic. In the first part of the presentation I will present the result of a double-slit interference experiment, which permits the measurement of the phase-shift of an electron traversing a quantum dot.



$FIG.1(a)$ $FIG.1(b)$

FIG. 1: (a) The scanning electron micrograph of the device. Electrons pass from a source (S) to a drain (D) through the two arms of a ring defined electrostatically by the gates (brighter shade). On the left-hand arm of the ring a quantum dot is formed by the two unmarked thin gates. In addition, another plunger gate (P) modulates the electrostatic potential of the dot and a variable magnetic field penetrates the inner core of the ring. (b) The schematic figure for the device.

The experimental system (see figure 1): The system is composed of three different regions, an emitter, a collector and a base region B in between. The emitter and collector constrictions are separated by a barrier with two openings (slits). One slit consists of the quantum dot whose behaviour we want to measure and the other is a reference slit. The number of electrons in the quantum dot is around $N_{el} \approx 200$. Now electrons pass from the emitter to the collector through the two slits of the ring defined electrostatically by the gates. The plunger gate modulates the electrostatic potential of the dot. A magnetic flux, Φ, threading the area enclosed by the two slits, results in an Aharonov-Bohm phase difference between the two interfering paths.

In the following parts of the presentation we will step by step understand all the experimental findings. We model the quantum dot as a double barrier system confining a well with quasi-bound states. The transmission amplitude through such a system shows resonances described by the Breit-Wigner formula. We will see that the observable Aharonov-Bohm phase difference for an applied magnetic field Φ is $\Delta\varphi = 2\pi\Phi/\Phi_0$, where $\Phi_0 = hc/e = 4.135 \cdot 10^{-7}$ Gauss $\cdot$ cm^2 is the fundamental unit of magnetic flux.

If the Aharonov-Bohm interference oscillations are recorded at many points along a conductance peak, a phase shift of the Aharonov-Bohm oscillations can be observed. The phase evolution shows a smooth and monotonic rise by almost π near the maximum of the resonance and the phase shift is on the scale of the resonance width. However, the sharp drop of the phase by π is also found in the tail of the peaks on a scale much smaller than the resonances width or any other energy scale available in the experiment. Thus, neighbouring resonances are in phase.

We will derive an relation (Friedel sum rule) between the phase shifts and the total charge Z of a potential

$$Z = \frac{2}{\pi} \sum_l (2l+1)\delta_l,$$

where $\delta_l(E)$ is the phase shift. In the case of the experiment we will see that the increment of phase and charge are related by $\Delta Q/e = \Delta\delta_l/\pi$. If we simpliy apply these rule to the case where the potential consists of the quantum dot, each addition of an electron to the dot requires an increase of δ_l by π. Thus, neighbouring resonances are expected to be off phase by π. But this is (as mentioned) not observed in the experiment.

Many investigations addressed the question of the abrupt phase changes. I will present a theory based on the Friedel sum rule. We will see that the Friedel sum rule is not strictly valid for quasi-1D systems. One consequence of the violation of the Friedel sum rule is that adjacent resonances can be either off phase by π or in phase. In the case of the experiment in-phase resonances can occur as generically as off-phase resonances.