Path integrals 4

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Up to now we have seen path integrals for bosons in real and imaginary time. In this case we expanded the propagator into an infinite product. By performing a Wick-rotation (that was the analytical continuation) we were able to apply the obtained results to statistical mechanics. One goal is to develop these techniques now for fermions. As you will see, it won't be neccesary to take the detour over real time, instead we will directly expand the partition function.

Unfortunately, it will not be possible to develop this formalism with c-numbers. To take into account Fermi-Dirac-Statistics we will introduce a new type of number, so-called Grassmann Numbers. With the introduction of an integral on these numbers, we can define coherent states in a very handy way. Analogous to the boson case we can then derive a closure relation which in turn enables us to expand the partition function as mentioned above. A technically important detail is that the only integrals we can solve are Gaussians.

We will apply this framework to a very simple yet relevant case of an electron gas. It will turn out that the Coulomb-two-body-interaction is not a Gaussian but has Grassmann numbers of fourth power in the exponent. Luckily there is a nice trick to circumvent the problem: The Hubbard-Stratonovich-Identity can bring it into the form of a gaussian at the cost of a new real path-intergral over φ. It is on this field φ that we will apply the Random Phase Approximation. This enables us to finally calculate some real physical properties of a metall. The given example is the formula of Gell-Mann and Brückner for the ground stateenergy.