Scattering Matrix Theory I

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The application of the scattering matrix theory to mesoscopic systems is presented in my talk. Situated between macroscopic and microscopic length scales, the spatial extent of mesoscopic systems ranges from a few nanometers up to several micrometers. To be specific, mesoscopic treatment is required whenever the size of a system becomes comparable to one or more of the following characteristic lengths: the de Broglie wavelength, the mean free path and the phase-relaxation length of an electron (at the Fermi energy) within the respective structure. Consequently, the wave nature of electrons plays an important role. The first part of the speech is dedicated to a derivation of the basic formalism that describes electronic transport in mesoscopic systems. Then this formalism is applied to a double barrier structure. Furthermore, the tunneling time of an electron through a potential barrier is analyzed.

In mesoscopic systems, in order to examine their electronic transport properties, the Landauer-Büttiker formula, in which the conductance through a sample is treated as a scattering problem, is widely used. In the two-terminal configuration the sample is connected to two reservoirs through a lead. The Landauer-Büttiker formula then describes the conductance through a sample by the transmission probability of an electron from one lead to the other.

In double-barrier structures two fundamentally different phenomena – arising from the wave nature of electrons (resonant tunneling) and from their particle nature (single-electron tunneling) respectively – can be observed. We concentrate on resonant tunneling. The conceptual framework provided by the scattering matrix theory allows us to consider the two barriers in series as individual scatterers. In the well between the barriers inelastic events are modeled by connecting to the well a conceptual side branch leading to a (fictitious) terminal. This terminal – treated as a real one – does not draw or supply a net current, but permits inelastic events and, therefore, phase randomization. We discuss the limiting regimes of completely coherent (no phase randomization) and completely incoherent (total phase randomization) tunneling, and also the continuous transition between the two. The overall results of our investigations will show that the peak value in transmission decreases with an increasing amount of inelastic scattering whereas the off-resonant transmission, simultaneously, increases.

Finally, we will focus on the problem of the tunneling times within a single-barrier configuration. In fact, there are several tunneling time definitions such as phase time, dwell time, Büttiker-Landauer time and local Larmor time; here, we show the consistency between two of them, i.e. the (bidirectional) phase time and the dwell time. The phase time measures the difference between the time it takes a wave packet peak to traverse an interval containing the barrier and the time it takes to traverse the same interval without barrier. Thus a phase time in transmission and reflection, respectively, is obtained. The bidirectional phase time can then be defined as the weighted sum of the latter two. The dwell time is a measure of the average time spent by particles in the barrier region. By extrapolation we evaluate the bidirectional phase time over an interval containing only the barrier region. A simple quantum mechanical calculation shows that the extrapolated bidirectional phase time is equal to the dwell time plus a self-interference term.