Optimization & Physics
Exercise 4

The goal of this exercise is to learn about hysteretic optimization.

Problem 4.1 Hysteretic Optimization

In this exercise we will apply hysteretic optimization to the Edwards-Anderson Ising spin glass. In the hysteretic optimization approach the energy is minimized by successive demagnetization. The Edwards-Anderson Ising spin glass Hamiltonian is extended by a field term and given by:

$$\mathcal{H} = -\sum_{\langle ij \rangle} J_{ij} S_i S_j - H \sum_i \xi_i S_i. \tag{1}$$

where H is an externally applied magnetic field and ξ a site-dependent direction of the field. The magnetization is generalized to

$$m = \frac{1}{N} \sum_{i=1}^{N} \xi_i S_i, \tag{2}$$

where N is the number of spins.
In this algorithm spins are updated if the local field h_i, which spin S_i is exposed to, shows in the opposite direction of S_i. The local field consists of the contribution of the neighboring spins and the site-dependent field:

$$h_i = \sum_{j \text{ neighbor of } i} J_{ij} S_j + H \xi_i. \tag{3}$$

Start with an external field $H = H_{\text{sat}}$ large enough, such that all spins S_i will show in the direction of ξ_i and update all spins, which are unstable. Decrease the external field by the value of the largest unstable local field and repeat this scheme until $-H_{\text{sat}}$ is reached and half of the hysteresis loop is done. Increase again the external field until you reach the newly defined saturation field $H_{sat} = \gamma H_{sat}$. A good choice is to set $\gamma = 0.9$. Make a plot of the magnetization as a function of the external field H. Run the algorithm for several instances of field patterns ξ_i and make a histogram of the energies found. Compare your results to the exact ground state energy you have obtained in the last exercises.