The Phases of Quantum Chromodynamics

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Overview

Introduction
- QFT and thermodynamics
- QCD and symmetries
- Simplifications of QCD

The QCD Phase Diagram
- Low temperature and finite density — The ground state \( T = 0 \)
- Low temperature and finite density — The situation for low temperatures
- Quark-gluon-plasma at high temperatures
- High temperature and large chemical potential

Colour superconductivity and colour flavour locking
- Symmetry breaking due to colour superconductivity
- Physical consequences of CFL

Relativistic Heavy Ion Collisions
Motivation and methods

▶ Motivation:
  ▶ High-temperature universe, i.e. fractions of a second after the Big Bang
  ▶ High-density matter, i.e. in a neutron star
  ▶ Understanding of QCD in extreme environments → deeper understanding of theory
Motivation and methods

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  - High-temperature universe, i.e. fractions of a second after the Big Bang
  - High-density matter, i.e. in a neutron star
  - Understanding of QCD in extreme environments → deeper understanding of theory

- **Methods:**
  - Almost no rigorous results
  - High temperatures, low densities: lattice calculations
  - High densities: analytic calculations
Finite temperature QFT

Short repetition of what has already been said for confinement/deconfinement:

- Partition function of a statistical system ($\beta = 1/T$):

$$Z = \sum_{\text{all states}} e^{-\beta E}$$
Finite temperature QFT

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  and Euclidian time $\tau = it$

- Final result:
  \[ Z = \int_{\phi(0)=\phi(\beta)} D\phi \exp \left[ - \int_{0}^{\beta} d\tau \int d^3x \mathcal{L}_E \right] \]
Chemical potential

- In canonical ensemble: particle number $N$ kept constant
Chemical potential

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- In a field theory:
  - Creation of particle-antiparticle pairs; conserved quantity defined through number of particles minus number of antiparticles
Chemical potential

- In canonical ensemble: particle number $N$ kept constant
- In a field theory:
  - Creation of particle-antiparticle pairs; conserved quantity defined through number of particles minus number of antiparticles
- More convenient: allow particle number to fluctuate, but introduce weight factor similar to Gibbs factor for the energy
Grand canonical ensemble

- Grand canonical ensemble: partition function defined by

\[ Z = \sum_{\text{all states } \alpha} e^{-\beta E_{\alpha}} e^{\beta \mu N_{\alpha}} \]
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- Grand (canonical) potential = Helmholtz free energy:

\[ \Omega(T, \mu) = -T \ln Z = -pV \]

- Minimized in equilibrium \( \rightarrow p \) maximized
Variables in the phase diagram

- Need to define driving parameters of the phase diagram
- Choose such that they are constant throughout the system even at phase coexistence and intensive
- One obvious variable: temperature $T$
- Density? No, because at phase coexistence, different density in different phases
- Chemical potential: connected to density, but constant across phase boundaries
Chiral symmetry: conventions

- Fermionic field described by Dirac equation
- Weyl representation used:

\[
\gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \psi(x) = \begin{pmatrix} \psi_L(x)' \\ \psi_R(x)' \end{pmatrix}
\]

\[
\psi_L(x) = \frac{1 - \gamma^5}{2} \psi(x)
\]

\[
\psi_R(x) = \frac{1 + \gamma^5}{2} \psi(x).
\]

- $\psi'_L$ and $\psi'_R$: left-handed and right-handed components
Chiral symmetry: conserved currents

Consider symmetry transformations:

\[ U(1)_B : \psi(x) \rightarrow e^{i\alpha} \psi(x) \]
\[ \psi(x) \rightarrow e^{i\alpha \gamma^5} \psi(x) \]

\[ \text{Divergences:} \quad \partial_\mu j^\mu = 0 \]
\[ \partial_\mu j^\mu_5 = 2 \quad \text{im} \psi \gamma^5 \psi \]

\[ \text{The associated currents are conserved!} \]
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\[ j^\mu(x) = \overline{\psi}(x)\gamma^\mu\psi(x) \]
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- The associated currents are conserved!
Chiral symmetry: Left- and right-handed currents

- Define left- and right-handed currents

\[ j^\mu_L = \overline{\psi_L} \gamma^\mu \psi_L \]
\[ j^\mu_R = \overline{\psi_R} \gamma^\mu \psi_R \]

- We find:

\[ j^\mu_L + j^\mu_R = j^\mu \]
\[ \partial_\mu j^\mu_L = \partial_\mu j^\mu_R = 0. \]

- Currents for left- and right-handed quarks are conserved separately!
Chiral symmetry in QCD

Consider QCD with a doublet of massless quark flavours:

\[ Q = \begin{pmatrix} u \\ d \end{pmatrix} = Q_L + Q_R, \quad Q_L/R = \frac{1 \mp \gamma^5}{2} \begin{pmatrix} u \\ d \end{pmatrix} \]
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- Transform separately under isospin transformations

\[ U_L, U_R \in SU(2) : \quad Q_L \to U_L Q_L, \quad Q_R \to U_R Q_R \]

- Chiral flavour symmetry of QCD: \( SU(N_f)_L \times SU(N_f)_R \), \( N_f \) number of (massless) quark flavours
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Full symmetry group of QCD:

\[ SU(3)_C \times SU(N_f)_L \times SU(N_f)_R \times U(1)_B \]
Chiral symmetry breaking in QCD

- Strong attractive interactions between quarks and antiquarks, i.e. negative contribution to total energy
- Energy cost to create a pair of massless particles is very small
Chiral symmetry breaking in QCD

- Strong attractive interactions between quarks and antiquarks, i.e. negative contribution to total energy
- Energy cost to create a pair of massless particles is very small
- Therefore, vacuum is populated by quark-antiquark pairs
- These have overall momentum and angular momentum of 0 → they carry net helicity charge, pairs of left-handed quarks and left-handed antiquark, which is antiparticle of right-handed quark
Chiral symmetry breaking in QCD 2

- Non-zero expectation value for quark-antiquark pairs:

\[ \langle 0 | \overline{Q} Q | 0 \rangle = \langle 0 | \overline{Q}_L Q_R + \overline{Q}_R Q_L | 0 \rangle \neq 0 \]

- Apply chiral flavour symmetries \( U_L, U_R \in SU(2) \):

\[ \langle 0 | \overline{Q}_L Q_R + \overline{Q}_R Q_L | 0 \rangle = \langle 0 | \overline{Q}_L U_L^\dagger U_R Q_R + \overline{Q}_R U_R^\dagger U_L Q_L | 0 \rangle \]
Chiral symmetry breaking in QCD 2

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  \]

- Only fulfilled for \( U_L = U_R \)!

- Appearance of condensate breaks
  \[
  SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V
  \]

- Spontaneous symmetry breaking: symmetry of Lagrangian not realized in ground state
Goldstone theorem

Assumptions:
- No long-range interactions, e.g. Coulomb forces → true for QCD w/o Coulomb
- Lagrangian has a continuous, global symmetry
- Potential term selects a ground state (minimum potential)
- Ground state does not respect symmetry
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Consequences (proof of classical, scalar case can be found in the report):
- Occurrence of a massless bosonic particle called (Nambu-)Goldstone boson
- True also for non-classical theories
Higgs mechanism

- Goldstone theorem: spontaneous breaking of a global continuous symmetry
- Higgs mechanism: spontaneous breaking of a local gauge symmetry!
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  - Lagrange function with local gauge symmetry
  - Ground state which is not invariant under gauge transformations
Higgs mechanism

- Goldstone theorem: spontaneous breaking of a global continuous symmetry
- Higgs mechanism: spontaneous breaking of a local gauge symmetry!
- Assumptions:
  - Lagrange function with local gauge symmetry
  - Ground state which is not invariant under gauge transformations
- Consequences:
  - New term in the Lagrangian: \( \Delta \mathcal{L} = \frac{1}{2} m_A^2 A_\mu A^\mu \)
  - Gauge bosons, described by \( A_\mu \), acquire mass!
Simplified QCD

- Electroweak interactions are ignored
- Two massless quarks $u$ and $d$, no other quarks
- Leads to global $SU(2)_L \times SU(2)_R \times U(1)_B$ symmetry, broken down to $SU(2)_V \times U(1)_B$
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The QCD Phase Diagram — schematically

- **Introduction**
- **The QCD Phase Diagram**
  - Low temperature and finite density — The ground state $T = 0$
  - Low temperature and finite density — The situation for low temperatures
  - Quark-gluon-plasma at high temperatures
  - High temperature and large chemical potential
- **Colour superconductivity and colour flavour locking**
- **Relativistic Heavy Ion Collisions**

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**The QCD Phase Diagram**

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**Chiral symmetry**

- **Restored (for $m_{u,d,s} = 0$):** $\langle \bar{\psi}\psi \rangle \approx 0$
- **Broken:** $\langle \bar{\psi}\psi \rangle > 0$

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**Crossover region for finite quark masses**

- **2nd order P.T. for zero quark masses**

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**1st order P.T.**

- **Exotic phases, e.g. colour superconductivity:** CFL
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The QCD Phase Diagram — schematically
Vacuum to nuclear matter: transition at $\mu_0$

- Consider partition function:
  $$Z = \sum_{\text{all states } \alpha} \exp \left( -\frac{E_\alpha - \mu N_\alpha}{T} \right)$$

- At $T = 0$: sum exponentially dominated by state which minimizes $E_\alpha - \mu N_\alpha$

- At $\mu = 0$: $N = E = 0$ with $n(\mu) = 0$
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- $\mu > 0$:
  - $E_\alpha - \mu N_\alpha > 0$: still $N = E = 0$ dominating
  - $E_\alpha - \mu N_\alpha \leq 0$: other states contribute

- Therefore expect phase transition to $n(\mu) > 0$ at
  \[ \mu_0 := \min_\alpha \left( \frac{E_\alpha}{N_\alpha} \right) \]

- $n(\mu)$ is order parameter for the transition
Vacuum to nuclear matter: value of $\mu_0$

- Problem: find value of $\mu_0$ for reduced and full QCD
- Reduced QCD:
  - Use $\frac{E}{N} = m_N - \frac{Nm_N - E}{N}$
  - Maximize second term $\epsilon = \frac{Nm_N - E}{N}$ which is binding energy per nucleon
  - Weizsaecker formula, w/o e.-m. interaction and for “infinitely large” nucleus: $\epsilon \approx 16 \text{ MeV}$
  - Find first-order phase transition to $n_0 \approx 0.16 \text{ fm}^{-3}$ at $\mu_0 \approx m_N - 16 \text{ MeV} \approx 923 \text{ MeV}$
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- Full QCD with Coulomb forces:
  - Infinite nucleus unstable due to Coulomb repulsion
  - Highest binding energy: iron nuclei $\rightarrow$ adding electrons for neutrality, phase transition to iron solid at

    $$\mu_0 \approx m_N - 8 \text{ MeV} \approx 931 \text{ MeV}$$
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The QCD Phase Diagram — schematically

- Chiral symmetry restored (for $m_{u,d,s} = 0$): $\langle \bar{\psi}\psi \rangle \approx 0$
- Chiral symmetry broken: $\langle \bar{\psi}\psi \rangle > 0$
- 1st order P.T.
- Exotic phases, e.g. colour superconductivity: CFL
- $\langle n_B \rangle = 0$
- $\langle n_B \rangle > 0$
- $\mu_0 = 923 \text{ MeV}$
- $\mu_1$
High-density phases

- $\mu_0 < \mu < \mu_0 + 200$ MeV: very little known
High-density phases

- $\mu_0 < \mu < \mu_0 + 200$ MeV: very little known
- $\mu \gg \mu_0 + 200$ MeV: particles occupy high momentum states due to Fermi statistics
- High momentum $\rightarrow$ asymptotic freedom $\rightarrow$ chiral condensate vanishes
- Restoration of chiral symmetry (exact for massless quarks) is accompanied with phase transition at $\mu = \mu_1$
- Speculation: more phase transitions, exotic phases: colour superconductivity
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The QCD Phase Diagram — schematically
The situation for low temperatures — Overview

- Asymptotic freedom for high momentum states still valid for finite temperatures
- Phase transition at $\mu = \mu_1$ not well understood, therefore little known about low-temperature behaviour
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- Asymptotic freedom for high momentum states still valid for finite temperatures
- Phase transition at $\mu = \mu_1$ not well understood, therefore little known about low-temperature behaviour
- At $\mu = \mu_0$:
  - $n(\mu) > 0$ for finite $T$ even for $\mu < \mu_0 \rightarrow$ no order parameter
  - But: discontinuities in 1st order phase transitions are assumed to appear as lines of 1st order p.t.s
  - First-order phase transitions should form lines terminated by critical point
The transition at $\mu = \mu_0$ for low temperatures

- Slope governed by Clausius-Clapeyron: $\frac{dT}{d\mu} = -\frac{\Delta n}{\Delta s}$
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  - Therefore $\Delta s = 0$ at $T = 0 \to$ infinite slope
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  - Therefore $\Delta s = 0$ at $T = 0 \rightarrow$ infinite slope
  - Gas (high $T$) expected to have lower particle density $\rightarrow \Delta n < 0$
  - At 1st order transition, system absorbs heat: $\delta Q < 0 \rightarrow \Delta s < 0$
  - With $\frac{\Delta n}{\Delta s} > 0$, we find
    $$\frac{dT}{d\mu} < 0$$
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  - With $\frac{\Delta n}{\Delta s} > 0$, we find

$$\frac{dT}{d\mu} < 0$$

- Expect line to terminate in critical point,
  $T_0 \approx \epsilon \approx 16$ MeV
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Chiral symmetry restored (for $m_{u,d,s} = 0$):
\[ \langle \bar{\psi}\psi \rangle \approx 0 \]

Chiral symmetry broken:
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1st order P.T.

Exotic phases, e.g. colour superconductivity; CFL

$\langle n_B \rangle = 0$

$\langle n_B \rangle > 0$

$\mu_0 = 923 \text{ MeV}$

$\mu_1$

$\mu$
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Low temperature and finite density — The ground state $T = 0$

$T_c = 166 \text{ MeV}$

Chiral symmetry restored (for $m_{u,d,s} = 0$):

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Crossover region for finite quark masses

2nd order P.T. for zero quark masses

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The QGP

- Raise temperature, keep chem. potential at $\mu = 0$
- Hadronic matter: dominated by pions as lightest mesons
- QGP: high-energy plasma of essentially free quarks and gluons, chiral symmetry restored
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  - Only pions formed in hadronic phase
The QGP

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- QGP: high-energy plasma of essentially free quarks and gluons, chiral symmetry restored
- Assumptions
  - Ignore interactions in hadronic phase
  - Only pions formed in hadronic phase
- Find transition: Helmholtz free energy $\Omega = -pV$ minimized
- Pressure is maximized $\rightarrow$ calculate pressure in the two phases
QGP transition: counting degrees of freedom

For non-interacting fields ($n_f$ number of degrees of freedom):

**Bosonic field** \(3P = \epsilon_B = n_f \frac{\pi^2}{30} T^4\)

**Fermionic field** \(3P = \epsilon_F = n_f \frac{7}{8} \frac{\pi^2}{30} T^4\)
QGP transition: counting degrees of freedom

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- We find

$$T_c \approx 150\text{MeV}$$
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- Not seen experimentally!
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Current view:

- Zero quark mass Second order
- Non-zero quark mass No chiral symmetry $\rightarrow$ no symmetry breaking $\rightarrow$ crossover region
Summary of known transitions

- Two lines of first order phase transitions along $T \approx 0$ and $\mu > 0$
- Second order p.t. (massless quarks) or crossover (finite quark masses) along $\mu \approx 0$ and $T > 0$
- Chiral condensate $\langle Q\bar{Q} \rangle$: vanishes in high-$T$- and high-$\mu$-phases
Summary of known transitions

- Two lines of first order phase transitions along $T \approx 0$ and $\mu > 0$
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- Chiral condensate $\langle Q\bar{Q} \rangle$: vanishes in high-$T$- and high-$\mu$-phases
- Chiral symmetry restoration in the region of high temperature and high chem. potential?
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Chiral symmetry restoration in the phase diagram

- **Massless quarks:**
  - Expect **one** line of phase transitions between regions of $\langle Q\bar{Q} \rangle = 0$ and $\langle Q\bar{Q} \rangle \neq 0$
  - Line of 2nd order p.t.s from the QGP transition and of 1st order p.t.s from the $\mu = \mu_1$ transition merge!
  - Meet in a tricritical point
Chiral symmetry restoration in the phase diagram

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- Finite quark masses:
  - Chiral symmetry is never exactly restored
  - Line of first order phase transitions from $\mu = \mu_1$ does not terminate in tricritical point
The Phases of Quantum Chromodynamics

The QCD Phase Diagram — schematically

Introduction

The QCD Phase Diagram

Low temperature and finite density — The ground state $T = 0$
Low temperature and finite density — The situation for low temperatures
Quark-gluon-plasma at high temperatures
High temperature and large chemical potential

Colour superconductivity and colour flavour locking

Relativistic Heavy Ion Collisions

- $T_c = 166\, MeV$
- Chiral symmetry restored (for $m_{u,d,s} = 0$): $\langle \bar{\psi} \psi \rangle \approx 0$
- 2nd order P.T. for zero quark masses
- 1st order P.T.

chiral symmetry broken: $\langle \bar{\psi} \psi \rangle > 0$
Colour superconductivity

- Mechanism proposed for low temperatures and high chemical potentials: asymptotic freedom at high densities
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Colour superconductivity and colour flavour locking
Symmetry breaking due to colour superconductivity
Physical consequences of CFL

Relativistic Heavy Ion Collisions

 Colour superconductivity

- Mechanism proposed for low temperatures and high chemical potentials: asymptotic freedom at high densities
- Two major types of colour superconductivity:
  - Two-flavour colour superconductivity (2SC) Two massless quark flavours
  - Colour flavour locking Three massless quarks flavour. Interest due to chiral symmetry breaking by other mechanism than chiral condensate.
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- Densities difficult to produce in earthly laboratories
- Only due to gravitational collapse: neutron star
Ordinary superconductivity

- Ordinary solid: ideal, non-interacting Fermi gas approximation becomes valid for small temperatures
- For some materials: at $T_c$, weak attractive force (mediated by ion lattice) leads to formation of condensate of Cooper pairs
- Described by BCS theory for type I superconductivity
- Physical effects:
  - Gap in the excitation spectrum: binding energy of Cooper pairs
  - Zero electrical resistance
  - Meissner-Ochsenfeld effect: expulsion of magnetic fields
Superconductivity in the QCD case

- Limit of high density: asymptotic freedom, expect free Fermi gas
Superconductivity in the QCD case

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- Attractive forces lead to formation of quark pair condensate:
  - Perturbative effects have small contribution
  - Non-perturbative (instanton) effects, proposed in late 1990s, have very large effect

\[ \text{Energy gap: } \Delta = E^2, \text{ } E \text{ binding energy} \]

\[ \text{Estimate: } \Delta \approx 100 \text{ MeV} \]
Superconductivity in the QCD case

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- Attractive forces lead to formation of quark pair condensate:
  - Perturbative effects have small contribution
  - Non-perturbative (instanton) effects, proposed in late 1990s, have very large effect
- Colour charge superconductivity and Meissner-Ochsenfeld are not observable
- Energy gap: $\Delta = \frac{E}{2}$, $E$ binding energy
- Estimate: $\Delta \approx 100$ MeV
Colour flavour locking

- Single-gluon exchange and non-perturbative effect provide attractive force between quarks
- Leads to formation of a condensate (analogous to Cooper pairs; \((\alpha, \beta)\) refer to colour, \((i, j)\) to flavour):

\[
\langle \psi_{iL}^{a\alpha} (\vec{p}) \psi_{jL}^{b\beta} (-\vec{p}) \epsilon_{ab} \rangle = -\langle \psi_{iR}^{a\alpha} (\vec{p}) \psi_{jR}^{b\beta} (-\vec{p}) \epsilon_{ab} \rangle \propto \epsilon^{\alpha\beta} A \epsilon_{ijA}
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  \]
- Locking of flavour and colour indices!
- Breaking of symmetries:
  \[
  SU(3)_c \times SU(3)_L \times SU(3)_R \rightarrow SU(3)_{\text{color},L,R}
  \]
- Chiral symmetry is broken, but by another mechanism
Pseudo-Goldstone bosons

- Spontaneous symmetry breaking:
  - Global symmetry: massless Goldstone boson
  - Local gauge symmetry: gauge bosons acquire mass
Pseudo-Goldstone bosons

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  - Colour symmetry broken $\rightarrow$ gluons acquire mass
  - Chiral symmetry broken: an octet of Goldstone bosons, oscillations of the diquark condensate
- If quark masses are non-zero, pseudo-Goldstone bosons are created with finite, but small, masses
- Can be considered as physical mesons with finite, computable masses: at $\mu = 400$ MeV, $m_{K^\pm} \approx 5 \ldots 20$ MeV
Neutron star cooling

- Neutron star:
  - Star between 1.44x and 3x mass of the sun
  - Density $\approx 10^{12}\text{kg/cm}^3$ in the core
  - Radius 10 km
Neutron star cooling

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- In CFL: all quarks acquire gap with $\Delta \gg T$, leading to low thermal excitation

- No contribution to specific heat from quarks, specific heat dominated by electrons and pseudo-Goldstone bosons
Objective: study QGP at high temperatures and low chemical potentials

Collide heavy nuclei (sulphur, lead, gold) with high energies (A: nucleus weight):

- Old: SPS @ CERN: CM energy 2A \ldots 18 A GeV
- Current: RHIC @ Brookhaven, NY: 200A GeV
- Future: LHC (ALICE) @ CERN: 5500A GeV

Other experiments will at some point attempt to study high-density matter, i.e. CBM/FAIR at GSI, Darmstadt
Stages of a RHIC collision

(a)

(b)

(c)

(d)
Thank you for your attention